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ELEMENTARY GRAPHIC STATICS

AND THE

CONSTRUCTION OF TRUSSED ROOFS.

A MANUAL OF THEORY AND PRACTICE.

BY

N. CLIFFORD RICKER, M. ARCH.,
Professor of Architecture, University of Illinois; Fellow of the American Institute of Architects, and of the Western Association of Architects.

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PREFACE.

A portion of this work appeared in the form of a series of articles on the Construction of Roofs, published in *Building*. These articles have been carefully revised, and greatly extended by the addition of such matter as appeared necessary to make the work more suitable as a manual for instruction, for private study, or for reference.

It now essentially represents the course of study in Graphic Statics, with special application to Trussed Roofs, pursued by the Students in the School of Architecture of this University for several years past, after a trial of the more favorably known text-books treating this subject.

The author has always believed, that, so far as possible, the student should receive full instruction in all those branches of the study which he will be required to apply in completely working out a design for a trussed roof, in actual practice.

Consequently, to determine the strains acting in the truss-members, to calculate the sectional dimensions required for these members, and to arrange the details of the connecting joints, embodying these details in suitable drawings, are all of equal and essential importance, though text-books usually stop with the first, leaving the student to acquire a knowledge of the others as best he may.

The author is not aware that any formulæ for determining the lengths of members of roof trusses have ever before been given.

That this little work may be found to substantially aid the diligent and inquiring student and draughtsman, as well as to serve as a work of reference for the architect, is the highest desire of the writer.

N. CLIFFORD RICKER.

UNIVERSITY OF ILLINOIS,
CHAMPAIGN, ILL., *July 24th*, 1885.

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CHAPTER I.

ELEMENTARY GRAPHIC STATICS.

DEFINITIONS.

In most architectural and engineering constructions, the different members of a structure are acted upon by various loads or pressures, but the structure is not usually moved thereby, because of the action of other pressures, by which the first are neutralized. Such structures and forces are then said to be in equilibrium, which is a state quite different from that in which no forces act on the structure.

(1.) *Statics* is a branch of Applied Mechanics, treating of the effects of forces in equilibrium, which neither produce motion, nor change the position of the body or structure acted upon.

(2.) *Graphic Statics* is a method of considering the mode of action and the effects of these forces, by means of a regular system of graphical operations, employed in place of mathematical computations, over which it possesses material advantages.

Discovered by Professor Culmann and perfected by later writers.

(3.) *A Force* is usually an influence or pressure exerted on one body by another body, by a fluid or gas, which may be at rest or in motion. Unless resisted or neutralized in some way, it causes the body to move along in a straight line, or changes its rate or direction of motion, if it be already moving.

(4.) A simple force always acts along a straight line, which is termed its *line of action*

(5.) The force may be assumed to be applied at any point of its line of action, this point being called the *point of application* of the force.

(6.) The magnitude or intensity of the force is always measured by some unit of weight, being expressed in pounds, tons, etc. The ton of 2,000 pounds is most convenient for this purpose, and will be employed hereafter unless otherwise noted.

Representation of a Force.

(7.) A given force may be fully represented by a right line, if the three following conditions are all satisfied:

a. Magnitude. Draw a straight line containing as many units of length as the given force contains units of force or weight. Any convenient scale of equal parts may be employed, though a decimally divided scale is most convenient.

b. Location. The line of action of the force must be either drawn or known, and the line representing the force must either coincide with or be parallel to this line of action.

c. Sense. The sense of a force is the direction in which it acts or tends to move the body affected, and must always be indicated, usually by an arrow-head attached to the line representing the force.

(8.) Example. Fig. 1.

Let AB be the line of action of a force F = 6 pounds, acting from B towards A.

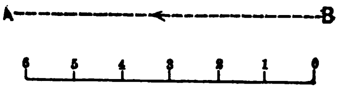

Fig. 1.
Representation of a Force.

Commencing at any point 0, draw a line of indefinite length parallel to AB. Lay off 01 equal to one unit of any convenient scale, here made one-fourth of an inch; make 06 equal to six times 01. Indicate the sense by an arrow-head. The given force F is then fully represented by the line 06, because the three prescribed conditions are all satisfied. (7.)

Resultant of several Forces.

(9.) All the forces are assumed to lie in a common plane, which coincides with the plane of the drawing or paper.

(10.) The resultant of two or more forces is that single force which would exactly replace them, and have the same effect on the body acted upon as the given forces.

(11.) The anti-resultant of the same forces would be that single force, which would exactly neutralize their effect and produce a state of equilibrium

(12.) Consequently, the resultant and anti-resultant of several given forces always have equal magnitudes and a common line of action, but their senses are opposed. If the given forces are already in equilibrium, they can have neither resultant nor anti-resultant. Otherwise, these can always be found, provided no two of the forces form a couple. (13.)

A Couple.

(13.) A couple is composed of two forces of equal magnitude, having parallel lines of action and opposed senses. It becomes evident that a couple tends to produce rotation of the plane of the two forces, and therefore it can neither be replaced nor neutralized by a single force. Hence, it can have neither resultant nor anti-resultant. A couple tends to rotate its own plane about any fixed point in this plane, termed its *centre of rotation.* If the direction of this rotation be like that of the hands of a watch, it is usually called *positive;* if in the opposite direction, *negative.*

(14.) Example. Fig. 2.

Let AB be the line of action of a force of 4 pounds, acting towards A; CD of an equal force acting towards D.

The given forces form a couple of negative rotation.

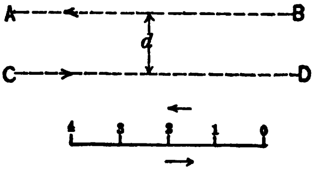

Fig. 2.
Representation of a Couple.

Commencing at any point 0, represent F1 by the line 04, and F2 will also be represented by 4 0. Indicating the senses, as in the figure, the couple is fully represented by the lines 04 and 4 0.

The perpendicular distance between the lines of action of the two forces is termed the *lever-arm* of the couple.

Components of a Force.

(15.) The components of a force are the two or more simple forces by which it may be replaced. Hence, a force is the resultant of its components.

Conditions of Equilibrium of Forces.

(16.) Forces are said to be in equilibrium if the following conditions are satisfied:

a. The given forces exactly neutralize each other, so that the position of the body acted upon is not changed.

b. They have neither resultant nor anti-resultant, nor form a couple.

c. Their force polygon must close.

d. Their equilibrium polygon must also close, its angles lying on the lines of action of the given forces. The truth of the last two conditions will become evident hereafter.

Composition of Forces.

(17.) Composition of forces signifies obtaining the resultant or anti-resultant of several given forces by combining them. This may be effected by the Parallelogram of Forces, or by the Force and Equilibrium Polygons.

a. By Parallelogram of Forces. Fig. 3.

(18.) Let two forces, F1, F2, act at any point A, along the lines BA and DA. Required, their resultant.

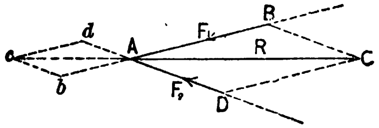

Fig. 3.
Parallelogram of Forces.

Represent F1 by AB, F2 by AD, and draw BC parallel to AD, and DC parallel to AB, thus completing the parallelogram ABCD. Join AC by the diagonal R, which represents the required resultant of the given forces, F1, F2. Its magnitude can be measured by applying the scale used in laying off F1 and F2 on AB and AD.

(19.) For, suppose a body to be placed at A, so arranged as to be free to move in any direction, but offering a uniformly increasing resistance to this motion, like that of a coiled spring. If the force F1 alone acts on this body, it would evidently move along the line AB produced, until it reached some point, *b*, at which the resistance becomes equal to the impelling force. Similarly, if F2 alone acts on the body at A, it must pass along AD produced, stopping at the point *d*, where the resistance equals F2. Now, if F1 be first applied, moving the body to *b*, and then F2 be allowed to act, it must pass along *bc*, which is parallel and equal to A*d*, stopping at *c*, where the resistance just equals the combined effect of the two forces. The same result would evidently be obtained if both forces were simultaneously applied to the body at A, or if their resultant R were applied instead.

For, ABCD and A*bcd* are similar parallelograms, because the angles, *d*A*b* and DAB are equal, and BA : A*b* :: DA : A*d*. Hence, AB : A*b* :: AC : A*c*. That is, the ratio of R to *ac* is the same as that of F1 to A*b* or of F2 to A*d*. Therefore, if the two forces are replaced by their resultant R, the same effect is produced as by the forces themselves. But their resultant R is AC, the diagonal of the parallelogram ABCD.

b. By Force Polygon. Fig. 4.

(20.) The lines of action of the given forces must pass through a common point of application A, and must also lie in a common plane.

Let the forces F1, F2, F3 and F4 act at the point A. Required, their resultant.

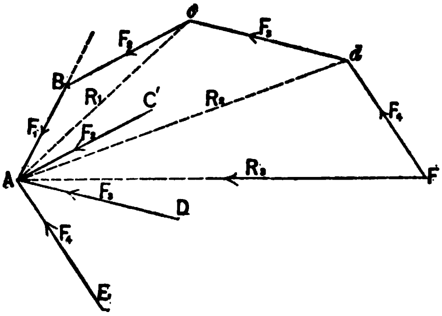

Fig. 4.
Force Polygon.

Represent F1 by AB, F2 by AC, F3 by AD and F4 by AE, severally laid off from A on their respective lines of action, to any convenient scale, and all having the same sense, toward or from A.

Commencing at B, draw B*c* parallel and equal to F2 or AC; *cd* parallel and equal to F3 or AD, and *d*F parallel and equal to F4 or AE; join AF, which is the required resultant of the four given forces, or, if its sense be reversed, it becomes their anti-resultant.

(21.) For, complete the parallelogram AB*c*C, and R1 or A*c* is evidently the resultant of F1 and F2; complete parallelogram A*cd*D, and R2 or A*d* will be the resultant of R1 and F3, or of F1, F2 and F3. Likewise, R3 is the resultant of R2 and F4, or of the given forces F1, F2, F3 and F4.

The polygon AB*cd*FA is termed the "*Force Polygon*" of the given forces F1—F4, because each of its sides represents one of the given forces, its last or "*closing*" side FA being their resultant R.

(22.) The following points should be carefully noted:

1. The forces must all have the *same* sense or direction around

the Force Polygon, excepting the resultant, whose sense is *opposed* to that of the forces.

2. The forces may be arranged in *any* order to form the polygon, provided each is taken but once, and with the proper sense.

3. If the sense of the resultant be reversed, it becomes their anti-resultant, with which they are in equilibrium. (11.)

4. Consequently, if the given forces are already in equilibrium, as is usually the case in structures, the force polygon must close, and all the forces composing it have the same sense. (12.)

5. The line of action of the resultant and anti-resultant will be the line AF.

The magnitude, location and sense of the resultant of several forces acting at a common point may therefore be fully determined by the Force Polygon.

c. By Force and Equilibrium Polygons.

(23.) The lines of action of the forces lie in a common plane, but do not intersect at a common point, the given forces having no common point of application.

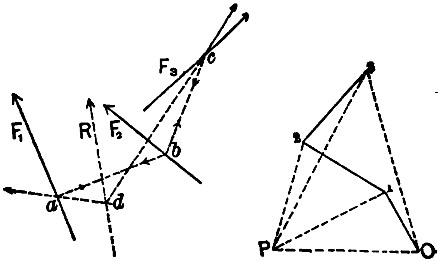

Fig. 5.
Force and Equilibrium Polygons.

Let the given forces be F1, F2 and F3, their lines of action being as shown in Fig. 5. Required, their resultant R.

Commencing at any point O, make O1 = F1, 1 2 = F2 and 2 3 = F3; join 3O. The polygon, O123O will then be the force polygon of the given forces, and its closing side 3O will be their resultant. (21.)

This resultant must have the same magnitude as 3O, to which it must likewise be parallel; it must have the sense from O towards 3, opposed to that of F1, F3. (22.) It is therefore fully determined by the force polygon, with the sole exception of the location of its line of action, which must be found by the equilibrium polygon.

(24.) Select any point P and join it with each angle of the force polygon by right lines, PO, P1, P2 and P3. Beginning at any point *a*, on the line of action of F1, draw an indefinite line parallel to PO, and *ab* parallel to P1, intersecting the line of action of F2 at *b*; draw *bc* parallel to P2, cutting F3 at *c*, and make *cd* parallel to P3. The lines *ad* and *cd* usually intersect at some point *d*, which is one point of the line of action of the resultant R, and which may then be drawn through *d* parallel to O3 of the force polygon. This fully determines the required resultant, since its magnitude, location and sense are all known. (7.)

(25.) For, suppose that the force F1 be applied at *a*, and that two other forces represented by PO and P1, respectively, act along the lines *da* and *ab*. These three forces, F1, PO and P1, form a triangle P, O, 1, consequently, if PO has the sense from *d* towards *a*, and P1 from *a* towards *b*, as indicated in the figure, their senses will be opposed to that of F1, which will then be their resultant. (22.) The force F1 may, therefore, be replaced at *a* by its two components, PO and P1, with the given senses.

Similarly, F2 may be replaced at *b* by two components, P1 and P2, acting along *ba* and *bc* as indicated; F3 may also be replaced at *c* by its components, P2 and P3, acting along *cb* and *dc*.

For the three original forces we have now substituted four others, whose effect is precisely the same as that of the given forces.

(26.) Now, the component of F1 acting along *ab* and that of F2 acting along *ba*, are each represented by P1, and they are therefore equal, but have opposed senses. Their sole effect would be to neutralize each other, and they may therefore be omitted without material error. In the same way, the component of F2 acting along *bc* and of F3 acting along *cb*, neutralize each other, and may be dropped.

Hence, the given forces are replaced by the two components not yet neutralized, one equal to PO and acting along *da*, the other equal to P3 and acting in *dc*. Their lines of action intersect at *d*, which is therefore one point of the line of action of their resultant. (18.)

But the resultant of PO and P3 is O3, which is identical with the resultant of the given forces F1, F2 and F3. The point *d* is therefore one point of the required line of action of the resultant of the given forces, which may then be drawn parallel to the closing side of the force polygon.

(27.) The same process and reasoning would be applicable to any number of given forces, since each force is replaced by its two components, and all these components neutralize each other, with the exception of the first and last, which intersect on the line of action of the resultant. Consequently, the method becomes perfectly general.

(28.) The point P is usually termed the "*Pole;*" the lines PO, P1, P2 and P3, joining the pole with the angles of the force polygon, are called "*Strings;*" the "*Force Diagram*" is the figure composed of the force polygon, the pole and the strings. The polygon *abcda* is called the "*Equilibrium Polygon,*" because if the senses of the resultant R and of the components acting along the sides of the equilibrium polygon *abcda* be reversed, the three forces applied at each of its angles would then be in equilibrium, and the polygon would not change its form.

(29.) Since the pole may be taken anywhere at pleasure, and the beginning point *a* may be chosen on the line of action of F1, it is evident that an infinite number of different force diagrams and equilibrium polygons may be drawn, but which all give the same resultant of the given forces.

This gives a means of checking the accuracy of the work, by taking a new pole and proceeding as before, obtaining another point *d'*, which must lie on the line of action drawn through *d*, if the work is correct.

Resolution of Forces.

(30.) This is the reverse of composition of forces, signifying the decomposition of a given force into two or more components by which it may be replaced or neutralized, if their senses are reversed. The lines of action of the two components must either both be parallel to that of the given force, or intersect it at a common point.

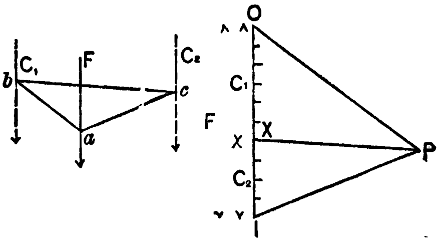

Fig. 6.
Resolution into Parallel Components.

Example. Components parallel to force. Fig. 6.

(31.) Let the lines of action of the components be C1 and C2 respectively, distant 2 feet and 3 feet from that of the given force F, to which they are parallel, and let F = 10 lbs.

Represent F by O1 (7); choose any pole P and join PO, P1. Commencing at any point *a* on the force F, draw *ab* parallel to PO and *ac* parallel to P1, intersecting C1 at *b* and C2 at *c*; join *bc* and draw P*x* parallel to *bc*. Then C1 = O*x* = 6 lbs., and C2 = *x*1 = 4 lbs. The components evidently vary inversely as their relative distances from the force F. (15.)

Example. Components not parallel to force F. Fig. 7.

(32.) Let the lines of action of the required components C1 and C2 intersect that of the force at *a*, and let F = 10 lbs.

Represent F by O1, and draw P1 parallel to C2, PO parallel to C1. Then C1 is represented by PO and C2 by P1, and their numerical values can be found by measurement with the scale used in laying off O1 = F. (15.)

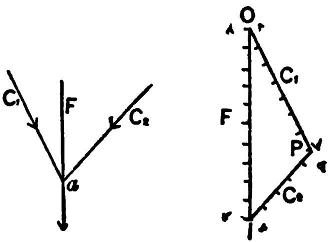

Fig 7.
Resolution into Components not Parallel.

Reactions at ends of a loaded beam or truss.

(33.) The lines of action of all the forces or loads supported by the beam lie in a common vertical plane passing through the axis of the beam, and may be parallel to each other or not. Their resultant may be found by (23) and (24), and may then be substituted for the forces. It may be resolved by (31) into two components parallel to itself, acting at the ends of the beam, and which are the downward pressures of the beam on its supports. If the senses of these components are then reversed, they will be the equal upward pressures of the supports against the ends of the beam, and which are in equilibrium with the original loads or forces supported by it. These will, therefore, be the required reactions. Their values are dependent on the loading, not on the form of the beam or truss.*

---

* This very useful method is due to Major J. R. Willett, of Chicago. (See *American Architect*, vol. III., pp. 30, 41, 55).

Example. Fig. 8.

(34.) Let the beam AB be acted upon by the four forces F1, F2, F3 and F4, which are assumed to be not parallel to each other, to make the case as general as possible. Required the reactions V1 and V2 at A and B.

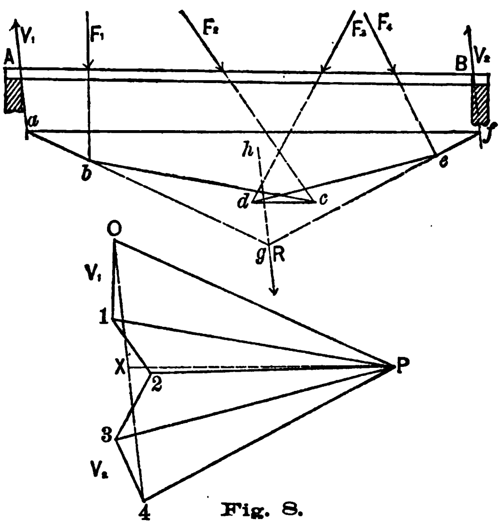

Fig. 8.

Reactions at ends of Beam.

Represent F1 by O1, F2 by 1 2, F3 by 2 3, and F4 by 3 4, forming the force polygon O-4; join O4, which will represent the resultant R of the four given forces. (20), (21.)

Take A and B on top of beam and over inner faces of the supports, and draw A*a* and B*f* parallel to O4, and which will be the lines of action of the required reactions V1 and V2, since these are to be parallel to the resultant R, which must be parallel to O4. (23), (24), (33.)

Choose any pole P and draw strings PO, P1, P2, P3 and P4. Commencing at any point *a* on A*a*, draw *ab* parallel to PO and intersecting F1 at *b*; then *bc* parallel to P1, *cd* parallel to P2, *de* parallel to P3, and *ef* parallel to P4, cutting B*f* at *f*. Join *af*, which is the closing line of the equilibrium polygon *abcdef*. Through P draw P*x* parallel to *af*, intersecting O4 at the dividing point *x*. Then O*x* =V1 and *x*4 =V2.

(35.) For, produce the first side *ab* and last side *ef* of the equilibrium polygon to intersect at *g*, and through *g* draw *gh* parallel to O4; *gh* will then be the line of action of the resultant R, by which the given forces may be replaced. (23), (24.) Let the four forces be replaced by their resultant R, and we then have the single force R, represented by O4, which is to be resolved into two parallel components V1 and V2. Then *ga* is parallel to PO, *gf* is parallel to P4, and P*x* to *af*, consequently O*x* =V1 and *x*4 =V2, whose senses must be opposed to that of R, since they represent the upward pressures of the supports.

(36.) If all the given forces are vertical and parallel, as is frequently the case when loads are supported by the beam or truss, the reactions V1 and V2 are likewise vertical and the force polygon becomes a vertical straight line, which is usually termed the "*load line.*"

If the pole P be taken on the left-hand side of the force polygon O-4, the equilibrium polygon will be inverted, that is, it will be convex upward, but this does not affect the values of V1 and V2, as these do not depend on the position of the pole P. This will frequently be found very convenient in obtaining the strains in roof trusses, since the paper below the truss diagram is then left free for working out the strain diagrams.

Reactions at Ends of Truss, one End Resting on Expansion Rollers.

(37.) Both ends of all short-span trusses, and of all trusses having wooden tie-beams, are usually firmly anchored to the walls of the building. But if the trusses are long and are built of iron, the stability of the walls would be seriously affected by the changes in the length of the trusses, resulting from the expansion and contraction of the metal, caused by changes in temperature. To avoid this danger, one end of a long iron truss usually rests on iron rollers, while the other is attached to the wall.

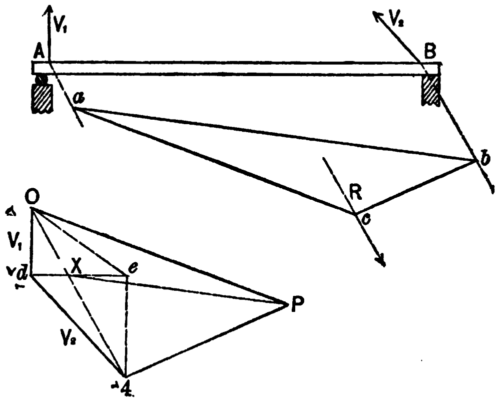

Fig. 9.
Reactions at ends of truss; one roller.

Consequently, if the friction of the rollers be neglected, the wall beneath them can only exert a vertical pressure on that end of the truss, *i. e.*, the reaction at that end must be vertical, while the remaining components of the resultant of all the forces acting on the truss must be supported at the fixed end. Hence, the two components or reactions V1 and V2 are then neither parallel to each other nor to the resultant R. (32.)

Example. Fig. 9.

(38.) Let AB represent a beam or truss of any form, the end A being supported by rollers placed on the wall, B being firmly fixed to the wall.

The original forces F1, F2, etc., which act on the beam, are omitted for the sake of simplicity, and it is assumed that they have been replaced by their resultant R, represented by O4, which has been found as in (35).

This resultant is then resolved by (32) into the two parallel components O*x* and *x*4, which would act along the lines A*a* and B*b*, were the roller omitted at A and both ends of the beam fastened to the walls. Through *x* draw the horizontal *de*, also the vertical O*d*, and join *d*4. Then V1 = O*d* = required vertical reaction at A, and V2 = *d*4 = required reaction at B. (32), (37.)

For V1 = O*d* = vertical component of *ox*, and *dx* = its horizontal component, which must be transferred through the beam and supported at B. V2 = the resultant *d*4 of *dx* and *x*4, and is therefore the required reaction at B. (10.)

(39.) Suppose that the end A be fixed and the roller be placed at B. Draw vertical 4*e* and join O*e*. Then V1 = O*e* and V2 = *e*4, which is vertical. (38.)

It is evident that rollers could not be placed at each end, since the beam would then roll off its supports, unless the resultant of the forces acting upon it were vertical.

MOMENTS OF FORCES.

DEFINITIONS.

(40.) The *Centre of Rotation* of the moment of a force is any fixed point, about which, as a centre, the force intends to rotate the plane containing both the point and itself.

(41.) The *Lever-Arm* of the moment of a force is the perpendicular, let fall from the centre of rotation on the line of action of the given force.

(42.) The *Moment of a Force* is the measure of its tendency or power to rotate its plane about the centre of rotation. The numerical value of the moment always equals the product of the magnitude of the force and the length of its lever-arm, and is expressed in foot-tons, inch-tons, inch-lbs., etc., according to the units of length and of force or weight employed. The foot-ton will here be used as the unit of moments, being equal to the effect of a force of one ton, with a lever-arm of one foot. (This term "*foot-ton*" is employed in a very different sense in mechanical engineering, to represent the force required to lift one ton one foot high in a minute, $=\frac{2}{33}$ of one horse power.)

(43.) A moment is said to be "*positive*" or "*negative*," according as it tends to produce rotation in the same direction as the hands of a watch, or the opposite.

Example. Fig. 10.

(44.) Let C be the centre of rotation and F the given force. Let fall the perpendicular C*a* on the line of action of the given force F; then C*a* is the lever-arm of the force, and F×C*a* = moment of F about C. This moment is negative, because it tends to produce rotation opposed to that of a watch.

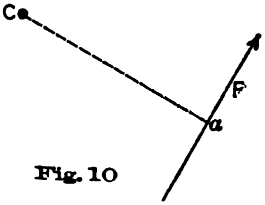

Fig. 10

Moment of a Force.

Moment of a Couple.

(45.) The *moment of a couple* (13) about any point whatever, in its own plane, equals the product of one force into the perpendicular distance between the lines of action of the couple. A couple can only be neutralized by another couple having an equal moment and an opposed rotation.

Example. Fig. 11.

(46.) For, let F1 and F2 form the given couple. Let x and $x1$ be the lever-arms of F1 and F2 about any centre of rotation C in their plane. The moment of F1 = F1×x; of F2 = F2×$x1$ the last being negative or minus, because its direction of rotation

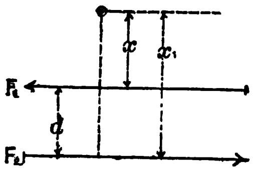

Fig. 11

Moment of a Couple.

is negative. The moment of the couple $= F(x - x1) = F \times d$, d being the perpendicular distance between the lines of action of F1 and F2. Since the same value would evidently be obtained for any other location of the centre of rotation C, the moment of a couple has a constant value $= F \times d$.

Resultant Moment of Several Forces.

(47.) This is that single moment, which would exactly replace the moments of all the given forces, or, if its direction were reversed, would exactly neutralize them and produce equilibrium, with any centre of rotation whatever. Hence, if several forces are in equilibrium, their resultant moment = 0. (12.)

Since the resultant moment of several forces is identical with the moment of their resultant about the same centre of rotation, it may be found by determining their resultant by (18), (20) or (23), when the moment of this resultant will be the required resultant moment.

Or, it may be found graphically by *Culmann's Method.*

Culmann's Method for a Single Force.

(48.) Let F (Fig. 12) be the given force and C the centre of rotation.

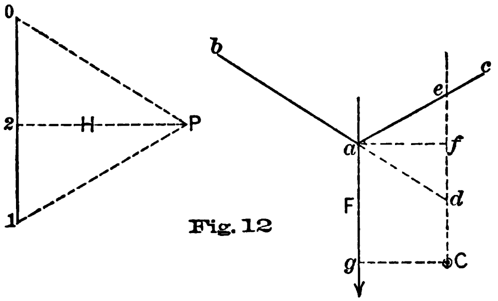

Fig. 12

Culmann's Principle.

Represent F by 01; with pole P, draw the strings P0, P1, and also draw P2 perpendicular to 01. The length of this perpendicular to 01 is usually represented by the symbol H.

Commencing at any point *a* on the line of action of F, draw the equilibrium polygon *bac*, by parallels to P0 and P1; produce *ba* indefinitely, and through C draw C*e* parallel to F or 01. Draw

Cg and af perpendicular to F or 01. Then the moment of F about $C = ed \times H$.

For, from similar triangles 01 P, dea, $01 : H :: de : af$. Hence, $01 \times af = de \times H$. But $01 = F$, and $af = Cg$, which is the lever-arm of F about C. Therefore, the moment of F about the centre of rotation $C = de \times H$.

The string P2 or H, perpendicular to 01, is termed the "*Pole Distance*," and the line de may be called the "*Intercept*," since it is a line drawn through C, parallel to F, and intercepted between the sides of the equilibrium polygon bac, which intersect on the line of action of the given force F.

Hence, *the moment of any force always equals the product of the corresponding intercept and the pole distance.* This is *Culmann's Principle*, which should be clearly understood, as it will be found very useful in the consideration of a loaded beam, purline, etc.

Culmann's Method for Several Forces, which may or may not be Parallel.

(49.) It has just been demonstrated that Culmann's method is true, when applied to a single force. (48.)

Since this single force may be the resultant of any number of given forces, it is evident that the principle becomes perfectly general, and is therefore applicable to any number of forces.

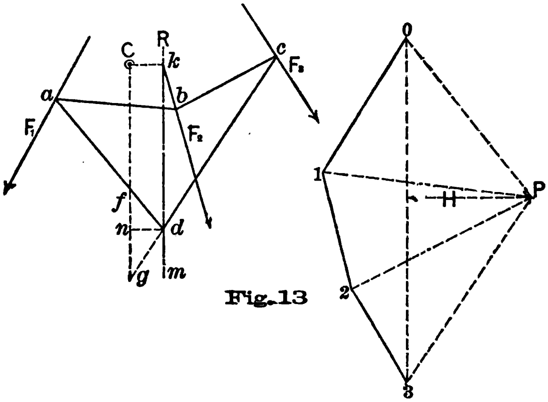

Fig. 13

Moment of several Forces.

Let F1, F2 and F3 (Fig. 13) be the given forces, C being any given centre of rotation.

loading be more complex, the graphical method is preferable, for it is easily applied to even the most complicated forms of loading, which are treated analytically with great difficulty, if at all.

A. By Formulæ.

Let W = total load on the beam, in net tons.

Let w = load on each lineal foot of the beam, in tons.

Let L = clear length of beam, in feet.

Let V = reaction at either support, in tons.

1. *Load concentrated at centre of beam.*

(52). $V=\frac{W}{2}$ = reaction at either end.

$\frac{Wx}{2}$ = bending moment at any point X, distant x feet from the left support, and lying between that and the centre.

$\frac{W\ (L-x)}{2}$ = bending moment for any point X between centre and right support.

$\frac{WL}{4}$ = maximum bending moment acting anywhere along the beam, and here found at its centre.

0 = bending moment at either end.

$\frac{W}{2}$ = shear at any point of the beam, except at the centre, where the shear theoretically $=0$.

2. *Load uniformly distributed.*

$V=\frac{wL}{2}=\frac{W}{2}$ = reaction at either end.

$\frac{wx\ (L-x)}{2}$ = bending moment at any point X, distant x feet from the left support.

$\frac{wL^2}{8}=\frac{WL}{8}$ = maximum bending moment acting on the beam, and found at its centre.

0 = bending moment at either end.

$\frac{w}{2}(L-2x)$ = shear at any point X.

$\frac{wL}{2} = \frac{W}{2} =$ maximum shear at either support.

$0 =$ shear at the centre of the beam.

B. Graphical Method.

1. *Load concentrated at several points.*

(53.) Let three loads, W1, W2 and W3, act on the beam at the points C, D and E. (Fig. 14.)

Find the reaction V1 at left, and V2 at right support, by employing the method explained in (33) and (34), using any pole P, and drawing the equilibrium polygon *abcde*. The "*dividing point*" is then at *x*, and V1 $= 0x$, V2 $= x3$.

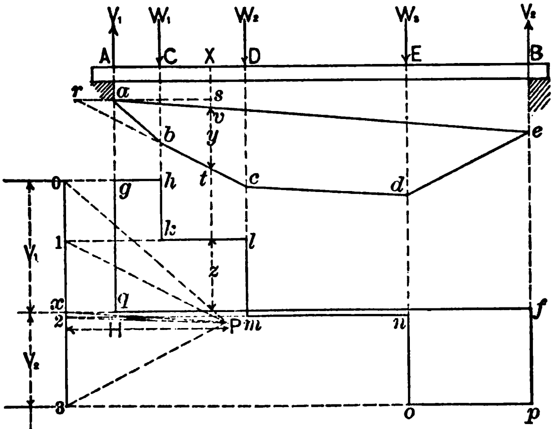

Fig. 14.
Several Concentrated Loads.

Through the dividing point *x*, draw the horizontal "*shear axis*" *xf*; draw the horizontal 0*h*, intersecting the vertical line of action of W1 in *h*; also 1 *kl* intersecting lines of action of W1 and W2 in *k* and *l*; then 2*mn*, cutting lines of action of W2 and W3 in *m* and *n*; lastly, 3*op*, intersecting line of action of W3 in *o*, and a vertical through the right support in *p*.

The broken line *ghklmnop* is termed the "*shear line*," and the diagram composed of the shear axis, shear line and the verticals drawn through the two supports, is called the "*Diagram of Shears*."

To determine the shear at any point X, draw a vertical through that point, and measure the length of that portion z of this vertical, comprised between the shear axis and the shear line, using the scale employed in laying off the force polygon 03. This will be the required shear at that point.

(54.) The equilibrium polygon *abcde* is also called the "*Diagram of Bending Moments.*"

To determine the bending moment at any point X, measure that portion of a vertical through the point included between the equilibrium polygon and its closing line *ae*, in feet, using the same scale as that of the length of the beam AB. Through the pole P draw a perpendicular to the resultant 03, which perpendicular is termed the "*pole distance*," and is usually represented by the symbol H. (48.) Measure the length of H in tons, at the same scale as that of 03.

Then $\mathrm{H} \times y =$ bending moment at X in foot-tons. (48.)

For, produce *ae* and *bc* to intersect at *r*, and draw the horizontal *rs*. Then from similar triangles *rvt*, $1x$P, we have $rs : y :: \mathrm{H} : 1x$. Hence, $rs \times 1x = \mathrm{H} \times y$.

But $1x$ is the resultant of V1 and W1, and *rs* is the lever-arm of this resultant for the point X; hence, $rs \times 1x =$ bending moment acting at X; hence $\mathrm{H} \times y =$ bending moment at X also.

If we call y the "*intercept*" for the point X, we obtain the following general rule:

The bending moment at any point of the length of a beam equals the product of the pole distance H and the corresponding intercept for the point (48.)

2. *Load continuous, not uniform.*

(55.) Let the loading be represented by the enclosed area ABCD (Fig. 15) above the beam, the vertical distance between the curve and the beam representing the relative intensity of the loading at the corresponding point of the beam.

Divide the area by verticals into strips of equal or unequal breadth; find the area and centre of gravity of each strip; substitute for the weight of each strip a numerically equal force, acting vertically through its centre of gravity; draw the equilibrium polygon or diagram of bending moments, and the shear diagram, in the way explained in the last case (54), (55).

(56*a*.) These diagrams are approximately, but not absolutely correct, for the loading is not actually concentrated at the centres of gravity of the strips, as assumed here. To correct this error, produce the verticals, which separate the strips, so as to intersect

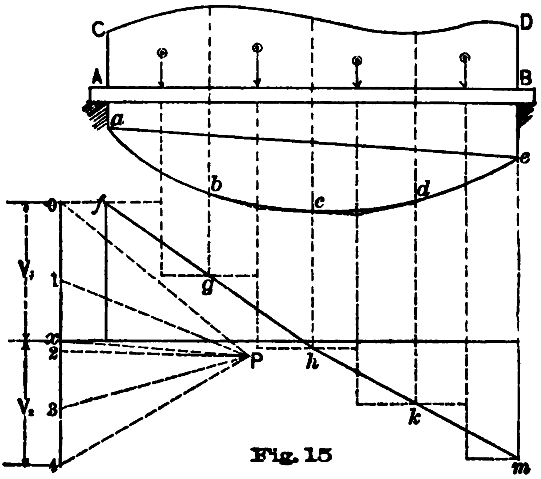

Fig. 15

Load continuous and varying.

the equilibrium polygon in the points *a*, *b*, *c*, *d*, *e*, and the shear line in the points *f*, *g*, *h*, *k* and *m*. Trace a curve tangent to the ends *a* and *e*, and the intersections *b*, *c* and *d* of the equilibrium polygon, which will be the "*Equilibrium Curve*;" the lengths of the intercepts are to be measured between this curve and the closing line *ae*. Trace a curve through the ends *f* and *m*, and the intersections *g*, *h* and *k* of the shear line, obtaining the "*Shear Curve*;" the true shears are to be measured between this curve and the shear axis.

General Deductions.

(57.) From examination and comparison of the preceding diagrams of bending moments and shears, we may deduce the following facts.

1. The maximum shear is always found at one end of a beam.

2. The zero shear is always found at or near the centre of the beam, at the same point as the intersection of the shear axis and the shear line of the shear diagram.

3. The maximum bending moment always occurs at or near the middle of the beam, at the same point as the longest intercept between the equilibrium polygon or curve and its closing line.

4. The zero bending moment is always found at each end of the beam.

5. The maximum shear and zero bending moment are always found at the same points, at one end of the beam.

6. The zero shear and maximum bending moment always occur together, at or near the middle of the beam.

The mode of computing the safe sectional dimensions of a beam, after its bending moments and shears are determined, will be given hereafter, in finding the sectional dimensions of principal rafters, purlines, etc. (Chapter VIII.)

CENTRE OF GRAVITY.

DEFINITIONS.

(58.) The centre of gravity of a plane figure is that point by which it may be suspended or balanced, without any tendency to rotation being produced, from any position.

An *axis of symmetry* is any right line, which divides the figure into two equal and similar parts. The centre of gravity of the figure is then found somewhere on this line. When two axes of symmetry can be drawn, making any angle with each other, the centre of gravity of the figure must always be at their intersection.

In case the two sets of parts into which the figure is divided are all equal and similar, the two axes are called "*axes of similar symmetry.*" (79.)

The centre of gravity of a regular geometrical figure is usually found at its geometrical centre.

Centre of Gravity of a Triangle.

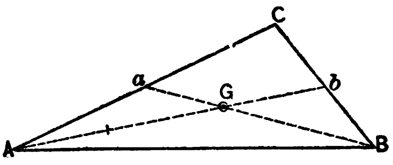

Fig. 16
C. G. of a Triangle.

(59.) Let ABC be the given triangle. (Fig. 16.)

Bisect any two sides as at *a* and *b*. Join each middle point with the opposite vertex of the triangle by right lines

B*a*, A*b*, intersecting at C, which will be the required centre of gravity of the triangle ABC.

(60.) Or, bisect one side, as at *b*; join *b* with the opposite vertex by the right line A*b*; which is then to be divided into three equal parts, the required centre of gravity being at that point of division nearest *b* or the base BC.

Centre of Gravity of a Trapezoid.

(61.) Let ABCD be the given trapezoid, the sides AB and CD being parallel. (Fig. 17.)

Bisect each parallel side, AB at *a*, CD at *b*, and join *ab*. Produce AB in either direction, making B*c* = CD; produce CD in the opposite direction, making C*d* = AB; join *dc*. The required centre of gravity of the trapezoid will then be found at G, the intersection of *ab* and *cd*.

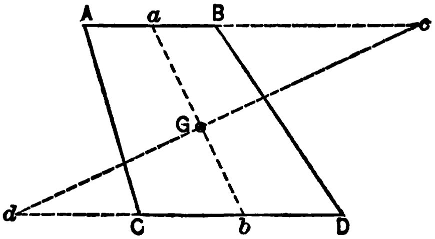

Fig. 17

C. G. of a Trapezoid.

Centre of Gravity of a Trapezium.

(62.) Let ABCD be the given trapezium, no pair of its sides being parallel. (Fig. 18.)

Divide the figure into two triangles by drawing either diagonal, as into the triangles ACB, CDB, by the line CB.

Find the centres of gravity of each triangle by (59) or (60), as at *g* and *h*; join *gh*.

Divide the trapezium into two other triangles ABD, ACD, by the other diagonal AD, and find their centres of gravity *f* and *e*; join *ef*. The required centre of gravity of the trapezium will be at C, the intersection of *ef* and *gh*.

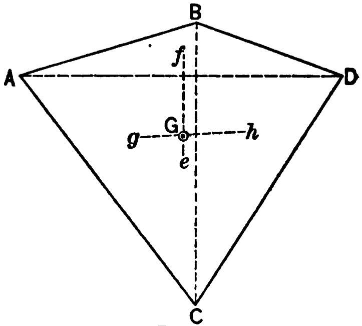

Fig. 18

C. G. of a Trapezium.

This method is also applicable to any quadrilateral.

Centre of Gravity of the Cross Section of a Beam or Column.

a. Two Axes of Symmetry.

(63.) The centre of gravity must be at the intersection of the two axes of symmetry AB and CD, Fig. 19. (58.)

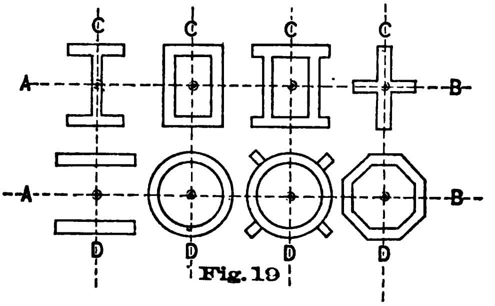

Fig. 19

Two Axes of Symmetry.

b. One Axis of Symmetry.

(63*a.*) The centre of gravity must lie somewhere on the axis AB. (58.) (Fig. 20.)

Divide the section into any number of strips of equal or unequal width by parallel lines, drawn perpendicular to the axis AB. Find the centre of gravity of each strip by (58), (59), (61) or (62), according to its shape; compute the area of each strip in square inches.

On any line 03, parallel to the lines subdividing the figure into strips, at any convenient scale, lay off 01 numerically equal to the area of the upper strip; 1 2 = area of second; 2 3 = area of the third, etc., in regular order from left to right, taking the strips in order from top to bottom of figure.

Draw P0 and P3, making angles of 45° with 03, and P will be the pole of the force polygon 03. (P must always be found in this manner and not assumed at pleasure, in this case.)

Assume that a force having a magnitude numerically equal to the area of each strip acts at right angles to the axis AB, through the centre of gravity of the strip. Then will 03 be the force polygon of these forces. (Fig. 20.)

The lines of action of the forces are parallel to 03, of course.

Commencing at any point *d*, on the line touching the top of the figure and parallel to 03, draw the equilibrium polygon *defgh*, producing its first and last sides to intersect at *k* (24); then draw *k*C parallel to 03, intersecting the axis AB at C, which will be the required centre of gravity of the figure.

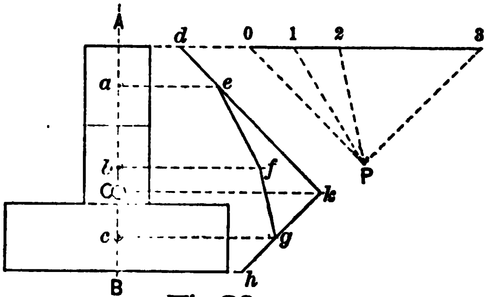

Fig. 20

One Axis of Symmetry.

(64.) For, it is evident that this centre of gravity may be found by experiment, by cutting out the full-sized section from any material of uniform thickness and weight, then balancing this over a straight knife-edge, at right angles to the axis AB. The weight of each strip is exactly proportional to its area, and may be replaced by a numerically equal force, acting at the centre of gravity of the strip, and perpendicular to the plane of the figure. The resultant of all these forces must pass through the required centre of gravity of the section. The equilibrium polygon of these forces is actually projected on the line AB, but for convenience it is revolved into the plane of the figure, to determine the point *k*, which is then revolved back to C on AB, becoming the centre of gravity of the given figure. The force polygon 03 may likewise be considered as being the force polygon of the forces replacing the areas of the strips, and revolved into the plane of the figure also.

c. No Axis of Symmetry.

(65.) Let the section have no axis of symmetry, as in Fig. 21.

Divide the figure into strips by horizontal parallel lines as before; replace areas of strips by numerically equal horizontal forces; draw force polygon *f*5 and equilibrium polygon *fghklmn*, determining the point *p*. (63), (64.) Draw horizontal line *p*G.

Assume that a vertical force, numerically equal to the area of each strip, acts at its centre of gravity; draw vertical lines of action of these forces; also, force polygon *t*8 and equilibrium polygon *tnvwx*, obtaining the point *z*; draw the vertical *z*G.

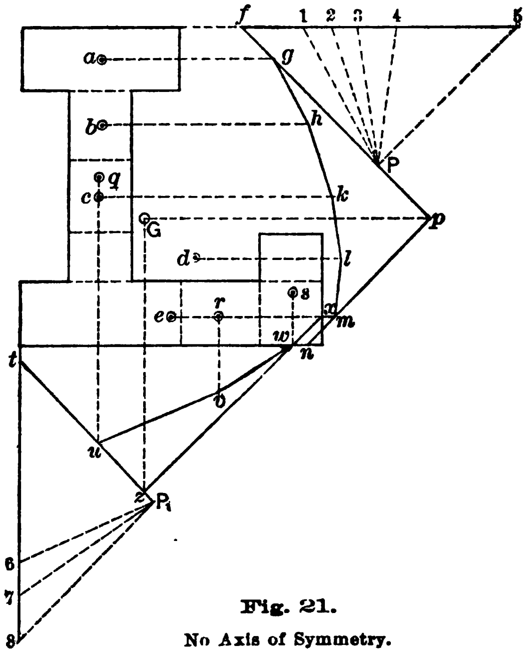

Fig. 21.

No Axis of Symmetry.

The required centre of gravity of the given section must lie somewhere on each of the two lines *p*G and *z*G, and will therefore be found at their intersection G. In the given case, the centre of gravity lies entirely outside the outline of the figure.

The methods of (63) and (65) are perfectly general, and are applicable to any form of plane figure, however irregular its outline may be.

MOMENT OF INERTIA.

DEFINITIONS.

(66.) The *Moment of Inertia* of a plane figure is a numerical quantity, whose value depends on both the form and the area of the figure, and which is always represented in formulæ by the symbol I.

When the plane figure is the cross section of a beam or of a long column, the strength and stiffness of the beam, and approximately that of the column also, vary directly as the moment of inertia of the figure; hence the evident necessity of a method for finding the value of the moment of inertia of any plane figure.

(67.) Let Fig. 22 represent any given plane figure; AB being any line or axis, drawn through the centre of gravity of the figure, the moment of inertia of the figure about the axis AB being required.

Take any minute square portion *a* of the figure, and multiply its area in square inches by the square of the perpendicular distance *y* from its centre to the given axis AB in inches, thus obtaining a numerical product.

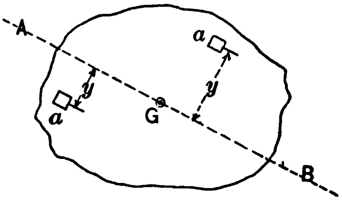

Fig. 22
Moment of Inertia.

Let this be done for each similar minute area *a*, into which the figure can be divided; take the sum of all the numerical products thus obtained, no matter whether the respective minute areas lie above or below AB. This sum will then be the required numerical value of the moment of inertia of the given figure about the axis AB only; the moment about any other axis might or might not be equal to that about AB. The accuracy of the method is evidently greatest when the area *a* is taken as small as possible. This method would be quite tedious and troublesome in practice, but is here given only for the purpose of clearly explaining the meaning of the term "*Moment of Inertia.*"

(68.) The *Radius of Gyration*, with reference to any axis AB, is that average value of the distance *y*, which would produce the same numerical value of the moment of inertia, as the actual and varying values of *y*.

Hence, the (radius of gyration)^2 × area of the figure = its moment of inertia.

$$\text{Or, radius of gyration} = \sqrt{\frac{\text{moment of inertia.}}{\text{area of figure.}}}$$

We will represent the radius of gyration by the symbol R*g* in the formulæ.

General Formulæ.

a. Axis passing through centre of gravity of figure.

(69.) Let $A =$ area of the given figure in square inches.

Let $I =$ its moment of inertia about the given axis.

Let $Rg =$ radius of gyration about the given axis in inches.

Then $Rg = \sqrt{\frac{I}{A}}$

And $A = \frac{I}{(Rg)^2}$.

Also, $I = A\ (Rg)^2$.

b. Axis not passing through the centre of gravity of the figure.

(70.) Let CD be the given axis, for which the moment of inertia of the given figure is required. (Fig. 23.) Through the centre of gravity of the figure, draw AB parallel to CD.

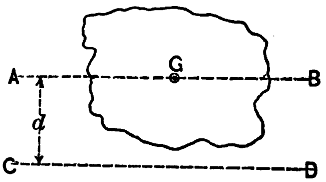

Fig. 23

Axis outside C. G. of Figure.

Let $I =$ moment of inertia for axis AB.

Let $Rg =$ radius of gyration for Axis AB.

Let $I' =$ moment of inertia for axis CD.

Let $Rg' =$ radius of gyration for axis CD.

Let $d =$ perpendicular distance between axes AB and CD in inches.

Then $I' = I + Ad^2 =$ moment of inertia for axis $\mathrm{CD} =$ moment of inertia for AB + product of area of figure into square of the perpendicular let fall from the centre of gravity of the figure on the given axis CD.

Also, $Rg' = \sqrt{Rg^2 + d^2} =$ radius of gyration for the axis CD.

Evidently I' and Rg' must always exceed I and Rg.

Formulæ for Moment of Inertia, etc.

(71.) The values of the moment of inertia, radius of gyration, etc., may be obtained by means of formulæ, in case of a few simple figures.

Let $d' =$ distance from the horizontal axis through the centre of gravity of the figure or section, to that fibre or part most distant from it, in inches. The other notation is indicated in Fig. 24. (*See page* 33.)

(72.) TABLE OF MOMENTS OF INERTIA.

SECTION.	AREA.	MOM. IN., I.	RAD. GYR. Rg.	d'
A	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{h}{4.243}$	$\frac{2h}{3}$
B	bh	$\frac{bh^3}{12}$	$\frac{h}{3.464}$	$\frac{h}{2}$
C	s^2	$\frac{s^4}{12}$	$\frac{s}{3.464}$	$\frac{s}{2}$
D	$bh-b_1h_1$	$\frac{bh^3-b_1h_1^3}{12}$	$\sqrt{\frac{I}{A}}$	$\frac{h}{2}$
E	"	"	"	"
F	$b(h-h_1)$	$\frac{b(h^3-h_1^3)}{12}$	"	"
G	πr^2	$\frac{\pi r^4}{4}$	$\frac{r}{2}$	r
H	$\pi(r^2-r_1^2)$	$\frac{\pi(r^4-r_1^4)}{4}$	$\frac{r^2+r_1^2}{4}$	r
I	$2bh+b_1h_1$	$\frac{2bh^3+b_1h_1^3}{12}$	$\sqrt{\frac{I}{A}}$	$\frac{h}{2}$
J	$bh-b_1h_1$	$\frac{bh^3-b_1h_1^3}{12}$	"	"
K	$bh+b_1h_1$	$\frac{bh^3+b_1h_1^3}{12}$	"	"
L	"	"	"	"

Graphical Method for Moment of Inertia, etc.

(73.) Let the given plain figure or cross section of beam be as represented in Fig 25, a form of section much used for cast-iron lintels.

The section has a vertical axis of symmetry AB, on which its centre of gravity must be found. (58.) Locate this centre of gravity by (63), or by (65), in case it has no axis of symmetry, by drawing the force polygon O4 and the equilibrium polygon *fghkmn-p*; then draw the horizontal *p*C, determining the required centre of gravity C. The line *p*C, produced across the

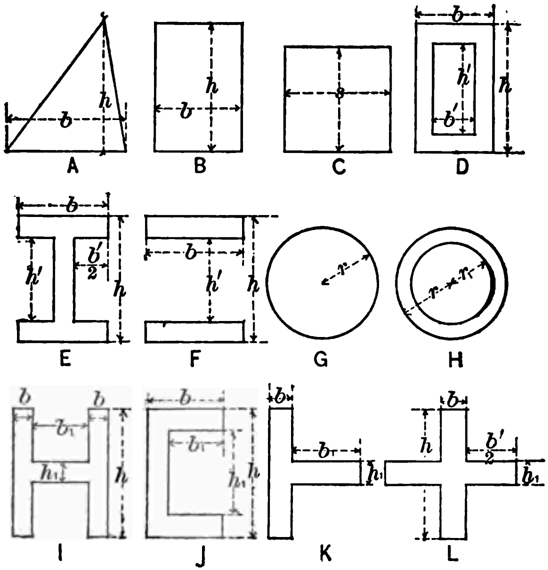

Fig. 24

Simple Figures.

section, must always be horizontal when the beam is set in place, and is usually called the "*Neutral Axis*" of the section, because the fibres of the beam, which lie in the neutral axis, are neither subject to compression nor tension, theoretically.

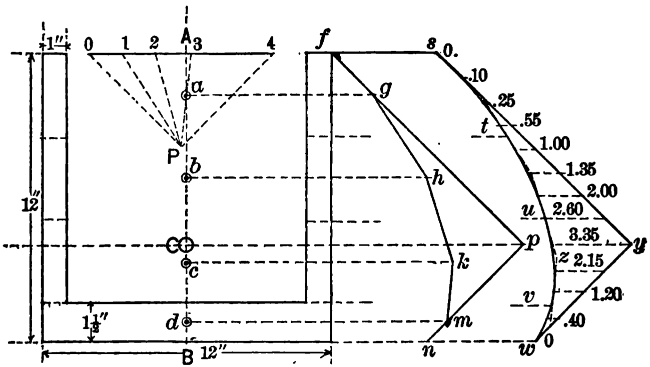

Fig. 25

Graphical Method for I.

(74.) After drawing the equilibrium polygon, as in Fig. 25, and finding the centre of gravity of the section, produce the horizontal lines, which separate the different strips into which the section is divided, to intersect the equilibrium polygon, as in the right hand portion of Fig. 25. Draw a continuous curve by means of a curved ruler, tangent to the polygon at its ends *s* and *w*, and also at each intersection *t*, *u* and *v*, just found. This curve may be termed the "*Equilibrium Curve*" and is to be substituted for the equilibrium polygon first found, because the curve corresponds to the division of the section into strips of infinitely small thickness. (56 *a.*)

(75.) The area comprised between the equilibrium curve *stuvw* and the tangents *sy* and *wy* at its ends, or the first and last sides of the equilibrium polygon produced, may be termed the "*Inertia Area*," and is to be found in square inches, that is, as it would be if the given section were drawn full size.

General Formulæ.

(76.) Let $A =$ area of given figure or cross section in square inches.

Let $A' =$ area of inertia figure in square inches, as if it were drawn full size.

Then $I = A \times A' =$ required moment of inertia of figure about the neutral axis pC. For any other axis, see (70).

Also, $R_g = \sqrt{A'} =$ radius of gyration for axis pC.

Finding Area of Inertia Figure.

a. By a Planimeter.

(77.) This is most nearly accurate and is easiest, since the area is found by merely passing the tracing point of the instrument around the perimeter of the inertia figure.

Let $\frac{1}{r} =$ ratio of reduction of scale of the given section from full size, $= \frac{1}{2}, \frac{1}{4}, \frac{1}{12}$, etc.

Measure actual area of inertia figure with a planimeter, set to read in square inches.

Then $A' = r^2 \times$ area obtained by planimeter.

b. By equidistant abscissas.

(78.) Commencing with the neutral axis pC or zy, draw a series of equidistant lines across the figure, parallel to pC, making the common distance x between any two adjacent lines as small as possible. Measure the length of each line, intercepted between the curve and its tangents, at the same scale as that of the given section, in inches, and write in these lengths, as in Fig. 25.

The upper and lower abscissas each $= 0$, but may fall outside the horizontal at top or bottom of the figure, though this involves no material error.

Then $A' = x \times$ sum of lengths of all the abscissas.

Also, $Rg = \sqrt{A'}$.

The accuracy of the result is greatest when the parallel lines are as close to each other as possible, but it will always be slightly larger than the true value. This error is eliminated by the use of the planimeter, but is usually quite small.

This graphical method for obtaining I, Rg and A' is perfectly general, practically accurate, and is easily applied to beam sections of complex form.

Axes of Similar Symmetry.

(79.) When two axes of symmetry may be drawn through the centre of gravity of a figure, which is thereby divided into exactly similar and equal portions in both cases, the moment of inertia of the given figure about any axis whatever passing through its centre of gravity will have a constant value, and the beam or column whose section is represented by the figure will be equally stiff in all directions. (58.)

CHAPTER II.

GENERAL CONSTRUCTION OF ROOFS.

DEFINITIONS.

1. *The Roof.*

(80.) A *Roof* is the covering or upper enclosing surface of a building, with the frame-work by which this is supported. It may be plane, cylindrical, spherical, etc.

A *Light Roof* is usually one of moderate span, without trusses, the rafters being directly supported by the walls or partitions of the building.

A *Heavy Roof* is employed for wider spans, and the rafters are then supported by purlines and trusses. It is usually required for spans of more than 20 feet.

The *Span* of a roof is the horizontal distance between the external surfaces of the walls of the building; its *Rise* is a vertical, let fall from its ridge to a horizontal line joining the intersections of the external surfaces of the walls and the roof surfaces; the *Inclination* of a roof equals the angle between its surface and a horizontal.

A *Bay* of a roof is that portion of its surface comprised between the vertical centre planes of two adjacent trusses; the same name is also sometimes applied to the space between the trusses themselv

A *Section Area* is that portion of the roof surface supported by a single purline or at a loaded point, and is the area comprised between the centre lines of two adjacent purlines and two trusses. The number of section areas supported by a single truss is usually one less than the number into which one bay is divided, or than the number of panels in a truss.

2. *The Truss.*

(81.) A *Truss* is a triangular, polygonal, or curved framework, whose ends rest on the walls. Its middle plane is vertical, and is at right angles to the walls, by which it is supported.

Any stable form of truss must always be composed of triangles, since the triangle is the only polygon whose form cannot be changed without altering the length of one or more of its sides.

The *Span* of a truss is the horizontal distance between the centres of its end-joints, and is usually the same as that between the centres of the walls, which support the truss; its *Rise* is the vertical connecting its span line and the centre of the joint at the apex or highest point of the truss.

The rise and span of a truss are evidently a little less than those of the corresponding roof surface.

A truss is frequently represented by a *Truss Diagram*, which is always composed of the centre lines of its members, and those meeting at any joint should always intersect at a common point, if possible. The truss diagram is drawn before commencing to find the strains on the different members of the truss.

A *Panel* of a truss is that portion lying between the centre lines of two adjacent vertical or radial members. Its form may be triangular, rectangular, trapezoidal, or that of a trapezium.

A *Member* of a truss is any straight or curved piece which connects two adjacent joints of the truss.

The *Upper Chord* is composed of the members which form the upper edge or margin of the truss. Each half of the upper chord of a triangular truss is often termed a *Principal.* The *Lower Chord* is composed of the members forming the lower edge of the truss. If straight, this is frequently termed the *Tie-beam* or *Tie-rod;* the first being a wooden timber, the second, one or more iron rods.

The *Web-members* connect the joints of one chord with those of the other, and may be radials in case of curved trusses, diagonals, or verticals. They may be *Struts*, capable of resisting compression; *Ties*, for tension only, *Strut-ties* or *Tie-struts*, for resisting either compression or tension alternately.

The upper chord is subject to compression; the lower, to tension only, as will be seen hereafter.

A *Joint* is the connection of two or more members, whose centre lines must intersect at a common point if possible, this common point being the centre of the joint. This is also sometimes called a *Vertex or Apex.*

A *Loaded Point* is a joint at which a load is attached to and supported by the truss. It is usually found on the upper chord only, at the points where the purlines rest on the chord, at the ends of two adjacent panels of the truss. The lower chord is not generally loaded, unless it supports a ceiling.

Mode of Supporting Roofs.

(82.) The rafters of light roofs are not trussed, but rest directly on the walls, and support the sheathing and covering of the roof. For spans of more than twenty feet the rafters may sometimes be placed two to three feet apart, and then trussed in pairs with strips of boards, etc.

Heavy roofs are supported by trusses resting on the side walls.

1. The sheathing is supported by rafters, which rest on the purlines, these being supported by the trusses.

2. The sheathing is supported directly by the purlines, the rafters being omitted.

SYNOPSIS OF A COMPLETE ROOF.

(83.) 1. *The Roof Surface.*
  a. The covering material.
  b. The sheathing or boarding.
  c. The internal ceiling, if any.

2. *The Supporting Frame-work.*
  a. The rafters.
  b. The purlines.

3. *The Trusses.*
  a. The upper chord or principals.
  b. The web-members; struts, ties and strut-ties.
  c. The lower chord; tie-beam or tie-rod.

CONSTRUCTION OF THE ROOF.

A. *Roof principally composed of wooden timbers.*

1. *The Roof Surface.*

(84.) The *Covering Material* protects the roof and the building from water, snow and wind, and may be composed of any impervious substance. Tin, sheet-iron, copper, lead, zinc, tiles, slates, shingles, etc., are used for this purpose.

The *Sheathing* is usually lumber of ordinary quality, one inch thick, either laid open for shingle, or laid close for tile and metal roofs. In good buildings a cheap grade of matched flooring is commonly used for the purpose, and it is sometimes made more nearly impervious to air and water by a covering of felt or heavy paper, placed beneath the covering material. If visible from beneath, the dressed surface of the lumber is turned downwards, and afterwards painted, stained and varnished, etc. A separate thickness of beaded wainscoting is sometimes placed below the sheathing of churches, and it is sometimes set diagonally.

In churches, the lath-and-plaster ceiling is frequently attached to the lower edges of the rafters, and must then be included in the loads supported by the roof and truss. In buildings for other purposes, the ceiling is usually supported by special ceiling joints and not by the rafters. This ceiling may also be composed of beaded wainscoting, plastering, building paper, etc.

2. *The Supporting Frame-work.*

(85.) The *Rafters* are usually scantlings, two to four inches thick and from four to twelve inches in depth, set edgewise, and placed at from twelve to twenty-four inches between centres. They are parallel to the upper chord of the truss or perpendicular to the edge of the roof, and are supported by the purlines, their feet resting on the wall-plates. The depths of rafters are sometimes increased towards their lower ends.

The *Purlines* are timbers nearly square in section, parallel to the edge of the roof, and, of course, are horizontal. They are placed from eight to sixteen feet apart, support the rafters, and are notched down on the upper chord to about one-half their depth at the loaded points.

The rafters are sometimes omitted, and the sheathing is then supported by the purlines, which are thinner and are placed from two to four feet apart. This system possesses some advantages in the construction of curved or cylindrical roofs, since the sheathing is then easily bent to the curve of the required surface, and it is not necessary to cut the rafters to the curve.

The upper surface of the purline is usually set parallel to the adjacent surface of the roof.

3. *The Truss.*

(86.) The *Upper Chord*, if straight, is composed of timbers of nearly square section, and from 16 to 24 feet long, spliced at or near the joints. For the sake of obtaining a good appearance, the upper chord is frequently made of uniform section throughout, though this requires more material. Being only subject to compression, simple halved splices are sufficient. If the upper chord be curved, it is usually built up from several thicknesses of 2, 3 or 4 inch plank, firmly spiked together, after being bent to the required curve. This method is often employed for building up straight chords also, because it is cheaper than splicing, the timbers are more thoroughly seasoned, and more readily obtained.

The *Lower Chord* or *Tie-beam* is a wooden timber, having the same horizontal breadth as the upper chord, to which its ends are firmly fastened. It is always subject to tension, and should be composed of timbers connected by strapped or fished splices. Or, it is more convenient and economical to build up the tie-beam from planks set edgewise and firmly spiked and bolted together. The last method is now frequently employed in good work because cheaper and better. If curved, the planks are laid flatwise, bent to the curve and fastened together, as described for the curved upper chord.

In some forms of trusses one or more tie-rods are substituted for the wooden tie-beam to obtain a lighter effect at an increased cost. The joints are then made by means of joint-pins, eyes being formed on the ends of the rods. The lengths of the rods are usually adjusted by sleeve-nuts or turn-buckles.

The *Web-members* are sometimes, in cheap trusses of small spans, composed entirely of strips of boards and scantlings.

The *Struts* are either timbers of square cross section, or their widths are the same as that of the upper and lower chords, so as to be flush with these on each face of the truss, for sake of appearance. The last method looks best, but requires somewhat more material than the first.

The *Ties* are usually round rods of wrought-iron, having nuts and washers on each end, because more convenient than if a head were formed on one end. Some material is saved by enlarging the ends before cutting the screw threads, but this is rarely done for small rods, or except in a large roof principally constructed of iron. Wooden ties are now very seldom used, because it is so difficult to properly connect them with the other members at the joints, except by the use of iron straps, bolts, etc., which makes them more expensive than rods.

The *Tie-struts* or *Strut-ties* are subject sometimes to compression, sometimes to tension, according to the forces and direction of the wind, and they are either composed of one or two timbers for resisting the compression, with a tie-rod for the tension, or a single timber may resist the compression, having straps, plates or bolts at each end, sufficiently strong to transmit the tension of the member to the chords.

The *Tie-rod* is substituted for the tie-beam, when the lower chord is to be of iron instead of wood, especially when it forms a broken line, the members then being more readily connected at the joints than if they were of wood.

Each member of the tie-rod is single, or it may be composed of two or more parallel, round or rectangular bars of wrought-iron, extending from joint to joint of the lower chord. These bars are connected to each other and to the web-members by means of eyes and cylindrical joint-pins, cast-iron sockets, etc., and their lengths are usually adjustable by sleeve-nuts or turn-buckles.

B. *Roof principally constructed of wrought-iron.*

This kind of roof is more fire-resisting than one of wood, and is therefore preferred for public buildings, fire-proof structures, and buildings exposed to special danger from fire.

1. *The Roof Surface.*

(87.) The *Roof Covering* is usually of tin, copper, sheet or corrugated iron, slates or tiles.

The *Sheathing* may be of boards, laid close or open; beneath it is sometimes placed a series of 4-inch brick arches, turned between iron beams, and brought to an external plane surface by concrete; or hollow tiles may be used, which make a warmer and lighter roof; or the sheathing may be entirely omitted, the slates or other roofing material then being attached to iron purline bars by copper wires.

2. *The Frame-work.*

(88.) The ***Purline Bars*** (if any) are small, horizontal, rectangular, T, I or channel bars, placed the same distance apart as the weathering of the courses of slates or tiles. They are fastened to the rafters by bolts or rivets and castings of proper forms.

The ***Rafters*** are larger T, I or channel bars, placed a few feet apart and parallel to the trusses; they are supported by the purlines, to which they are bolted or riveted; sometimes omitted.

The ***Purlines*** are usually T, I or channel bars of considerable size, supporting the rafters and fastened to the upper chord at the loaded points, which are eight to sixteen feet apart. If the rafters are omitted, as is usually the case in curved roofs, the purlines are then placed three or four feet apart, and are attached to the upper chord by castings of different heights, so that their upper edges are properly brought up to the desired curve. Arches of bricks or hollow tiles are sometimes then turned between the purlines.

3. *The Truss.*

(89.) The *Upper Chord* is a single bar, or is composed of two or more T, I or channel bars, firmly connected together by lacing bars or plates, the splices being made at or near the joints, by means of riveted patch plates. For roofs of wide spans, channel bars are commonly employed, their top and bottom flanges being turned outward and laced together by diagonal bars. For curved roofs, the upper chord is usually polygonal in form, each member being straight and not bent to the curve, the purlines being set out to the curve by joint-blocks of proper height.

Struts are either single I or star bars, or they are built up of two Ts, two channels, or of four angle bars riveted together, and having eyes forged on their ends, so as to connect with the adjacent members by pin joints. Riveted joints made with patch plates are common in Europe, but are rare in the United States.

Ties are either round iron rods with nuts and washers or with eyes, or they may be rectangular bars with eyes; they are frequently adjustable by turn-buckles, etc.

Tie-struts and *Strut-ties* are similar to struts, being so arranged as to resist either compression or tension.

The *Lower Chord* is always composed of wrought-iron rods or rectangular bars, connected at the ends by eyes and pins.

(90.) The different members of iron trusses are connected at the joints in one of two ways.

1. *By Patch Plates and Rivets.* Rarely used in the United States, except for connecting the members of the upper chord.

2. *By Pins and Eyes.* This is usually preferred, because the trusses are more easily and quickly erected, requiring less manual labor and scaffolding, and the strains in the members can be more accurately determined than when rivets and patch plates are employed.

CHAPTER III.

LOADS AND PRESSURES ON ROOFS.

(91.) These are taken per square foot of the inclined surface of the roof, except where otherwise stated. They are of two kinds: *Permanent,* which act constantly after the completion of the roof; and *Temporary,* which only occur at irregular intervals and act during limited periods.

1. *Permanent Loads.*

(92.) *a. Roof covering only.*
Shingles, 16 inch, 2 lbs.
Shingles, long, 3 lbs.
Tin and paint, 1 lb.
Iron, sheet and paint, 1½ lbs.
Iron, galvanized, 1 to 3 lbs.
Iron, corrugated, 1 to 3¾ lbs.
Copper, sheet, ¾ to 1¼ lbs.
Zinc, 1 to 2 lbs.
Felt and asphalt, 1 lb.
Felt and gravel, 8 to 10 lbs.
Slates, average, 10 lbs.
Tiles, plain, average, 12 lbs.
Tiles, fancy, laid in mortar, 25 to 30 lbs.

(93.) *b. Sheathing per square foot.*
Pine, hemlock, spruce, poplar, redwood, per inch thick, 3 lbs.
Chestnut or maple, 4 lbs.
Ash, hickory, Georgia pine, oak, 5 lbs.
Brick arches 4 inches thick and concrete, 70 lbs.
Porous tiles for slating, without slates, 10 lbs.
Hollow tiles, 3¾ in. flat, 12 lbs.
Hollow tiles, 6 in. arches, 22 lbs.
Hollow tiles, 9 in. arches, 32 lbs.
Hollow tiles, 12 in. arches, 36 lbs.

(94.) *c. Rafters, per square foot of roof.*

White pine, 2×4, 16 in. centres, 1.5 lbs.

White pine, 2×6, 16 in. centres, 2.25 lbs.

White pine, 2×8, 16 in. centres, 3 lbs., etc.

For heavier woods, increase these weights proportionally, or determine dimensions of rafters by formulæ for rafters, and then compute their average weight per square foot of roof, allowing the same weights per square foot of board measure already given for sheathing. (93.)

For purline bars and rafters of wrought-iron, first determine their sectional dimensions by the proper formulæ; their weights are then easily computed by allowing 3⅓ lbs. per lineal foot of bar per square inch in its cross section. Then compute average weight per square foot of roof surface.

(95.) *d. Purlines.*

Approximate weight per square foot of roof surface, if of white pine, other woods in proportion. (93.)

1. If supporting rafters, 1 to 3 lbs.
2. If supporting sheathing, no rafters, 2 to 4 lbs.

Or, compute dimensions of purlines by formulæ therefor, then finding their weight per square foot as in (94).

For iron purlines, proceed as for iron rafters. (94.)

Approximate weight of iron purlines is from 2 to 4 lbs. per square foot of the horizontal projection of roof surface.

(96.) *e. Ceiling, if any be used.*

Wainscoting, same as sheathing of equal thickness. (93.)

Lathing and plastering, 2 coats, 9 lbs.

Lathing and plastering, 3 coats, 10 lbs.

Brick arches or hollow tiles, same as for sheathing. (93.)

Light ceiling tiles, supported by T iron joists, without plastering, 5 lbs.

(97.) *f. Truss, per square foot of area covered by roof.*

The weight of the truss varies with the span and inclination of the roof, the distances between adjacent trusses, the kind of materials used in its construction, etc.

But in practice, the approximate weight of the truss is computed by first assuming its average weight per square foot of the horizontal projection of the roof, or the area covered by it, then multiplying this weight by the horizontal projection of that portion of the roof actually supported by the truss.

The following table is constructed from data given by different authorities, and gives the approximate weight of truss per square foot of horizontal projection of the roof, for trusses constructed of wooden timbers and iron rods, and also for trusses entirely constructed of wrought-iron. It will be noticed that the weights of trusses of the latter type are considerably greater than those of the former.

TABLE OF WEIGHTS OF TRUSSES.

SPAN.	WOODEN.	IRON.	SPAN.	WOODEN.	IRON.
Ft.	Lbs.	Lbs.	Ft.	Lbs.	Lbs.
10	.60	.92	140	7.40	12.00
20	1.20	1.83	150	8.00	12.55
30	1.82	2.75	160	8.50	13.15
40	2.10	3.75	170	9.00	13.70
50	2.50	4.63	180	9.50	14.27
60	3.10	5.50	190	10.00	14.85
70	3.70	6.38	200	10.50	15.42
80	4.25	7.38	210	11.00	16.00
90	4.75	8.28	220	11.50	16.58
100	5.25	9.00	230	12.00	17.15
110	5.75	9.85	240	12.50	17.75
120	6.35	10.75	250	13.00	18.30
130	6.80	11.35			

The weights for spans intermediate between those given in the table can easily be found by a simple interpolation between the two nearest given values.

The span of a truss is very seldom required to exceed 250 feet.

(98.) The actual weight of any required truss is determined as follows: Assume the weight of the truss according to the table; determine the strains in the members of the truss by methods to be given hereafter; then find dimensions required for the sections of these members, according to the material of which they are composed; finally, compute their weights, whose sum will be the corresponding actual weight of the truss; divide this total weight by the horizontal area covered by one bay of the roof, *i. e.*, the roof supported by one truss, and the quotient will be the weight of this truss per square foot of horizontal area. If this weight differs materially from that assumed from the table, then take it as the true weight for the truss, and repeat the process until the assumed and computed weights practically accord, and these will be the required actual weight.

Let $w =$ weight of the truss per square foot of horizontal area covered by the roof in pounds.

Let $i =$ angle of inclination of the surface of the roof.

Then $w \cos. i =$ weight of the truss per square foot of inclined roof surface.

2. *Temporary loads.*

The maximum intensities of the temporary loads rarely or perhaps never occur. Their values are assumed with reference to the results of experiment and observation, so as to be safe under all circumstances.

(99.) *a. Snow.*

Weight per square foot of horizontal area covered by the roof.

The weight of freshly fallen snow is about one-eighth that of water, or averages 8 lbs. per foot in depth. If mixed with hail or sleet, it may weigh four times as much, or 32 lbs. per foot. But its depth then rarely exceeds a few inches. The following table is believed to make a sufficient allowance.

TABLE OF MAXIMUM WEIGHT OF SNOW PER SQUARE FOOT.

Northern New England, New York, Michigan, Minnesota...	**30 lbs.**
Boston, Albany, Buffalo, Milwaukee, St. Paul..............	**25 "**
New York City, Cleveland, Chicago, Des Moines............	**20 "**
Philadelphia, Pittsburg, Wheeling..........................	**15 "**
Baltimore, Cincinnati, Indianapolis........................	**10 "**
Richmond, Louisville, St. Louis............................	**5 "**

In sheltered mountain valleys, the snow usually falls to a great depth, which must be determined and considered in designing structures for such localities.

The weight of snow per square foot of the inclined roof surface may easily be found by multiplying the given weight by the cosine of the angle of inclination.

(100.) *b. Wind pressure.*

The intensity of the pressure of the wind on a roof evidently varies with its inclination, but the relation of the two values is not accurately known. The direction of the wind is usually horizontal, causing a practically uniform pressure, perpendicular or normal to the roof. Its maximum velocity is here taken at 100 miles per hour, which produces a pressure of about 50 lbs. per square foot of a plane surface, placed at right angles to its direction.

Hutton's formula is generally employed by American and English architects and engineers, and will be used here in lieu of a better. It probably gives values somewhat larger than the true ones, and is therefore safe, though causing the use of a slight excess of material in the truss.

Let P = maximum pressure on a vertical plane surface in lbs. per square foot.

Let Pn = maximum pressure acting perpendicular to surface of the roof, in lbs. per square foot.

P is usually taken at 40 lbs. for buildings in protected situations, and at 50 lbs. for those on exposed sites.

It will be best to take the larger value for buildings erected in the Western States, in localities subject to violent winds. The smaller value will suffice for buildings of ordinary size in cities, which are usually sheltered in part.

Then

$$P^n = P \sin. i^{1.84 \cos. i - 1.}$$

The table on the opposite page is based on this formula, taking P at 40 lbs. and at 50 lbs.

(101.) TABLE OF NORMAL WIND PRESSURES.

Inclination.	Pressure—40 lbs.	Pressure—50 lbs.	Inclination.	Pressure—40 lbs.	Pressure—50 lbs.
Degrees.	Lbs.	Lbs.	Degrees.	Lbs.	Lbs.
1	1.2	1.5	31	27.2	34.0
2	2.2	2.7	32	28.0	35.0
3	3.1	3.9	33	28.7	35.9
4	4.1	5.1	34	29.5	36.8
5	5.1	6.4	35	30.1	37.6
6	6.0	7.5	36	30.8	38.5
7	6.9	8.6	37	31.5	39.3
8	7.8	9.7	38	32.2	40.2
9	8.7	10.9	39	32.8	40.9
10	9.6	12.0	40	33.3	41.7
11	10.6	13.2	41	33.9	42.4
12	11.4	14.3	42	34.5	43.1
13	12.4	15.5	43	35.0	43.8
14	13.3	16.6	44	35.6	44.5
15	14.2	17.7	45	36.0	45.0
16	15.0	18.8	46	36.5	45.6
17	15.9	19.9	47	37.0	46.2
18	16.8	21.0	48	37.3	46.7
19	17.6	22.0	49	37.9	47.3
20	18.4	23.0	50	38.1	47.6
21	19.3	24.1	51	38.4	48.0
22	20.2	25.2	52	38.7	48.4
23	21.0	26.2	53	38.9	48.7
24	21.8	27.2	54	39.2	49.0
25	22.6	28.3	55	39.4	49.3
26	23.4	29.2	56	39.6	49.5
27	24.2	30.2	57	39.7	49.7
28	25.0	31.3	58	39.8	49.8
29	25.7	32.1	59	39.9	49.9
30	26.5	33.1	60	40.0	50.0

For angles between 60 and 90, same as for 60.

CHAPTER IV.

RITTER'S METHOD OF MOMENTS.

(102.) This method for determining the magnitudes of the strains acting in the members of a roof truss is next best to the graphical one, requiring less drawing, but necessitating considerable arithmetical computation. Still, it is often very useful for checking the accuracy of other methods, since it is readily applied to a few members of the truss, and the results may then be compared with those previously obtained. It will here be applied to a simple form of truss only, but the effect of inclined forces will be considered, which has not before been done, so far as the writer is aware. For a complete exposition of this method, see Ritter's "Iron Bridges and Roofs," English translation by Sankey.

Moment of a Force.

(103.) Let B be any fixed point, lying, of course, in the plane of the force F, Fig. 26. Through B draw BA perpendicular to the line of action of the force.

The fixed point B is termed the centre of rotation of the force F, or of its plane. (40.)

The perpendicular BA is the "lever-arm" of the force, with reference to B. (40.)

The "moment" of a force always equals the product of its magnitude and its lever-arm, here $= F \times BA$.

This moment may be expressed in inch-lbs., foot-tons, etc., according as the magnitude of the force is measured in lbs. or tons, and its lever-arm in inches or feet. The term foot-ton merely represents the moment or effect of one ton with a lever-arm one foot long.

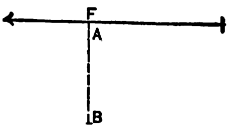

Fig. 26.
Moment of a Force.

It is evident that the force may cause its plane to rotate about the centre B to the right, like the hands of a watch, or to the left, as in Fig. 26. The former is usually termed "*positive*," and the latter "*negative*," and the signs + and − are prefixed to the corresponding moments. (42.)

Equilibrium of Moments.

(104.) If several forces act in a common plane, and are also in equilibrium, their moments about any centre of rotation in that plane must be in equilibrium likewise. (16.) That is, there will be no tendency of their plane to rotate in either direction, and the algebraic sum of their moments $=0$. (47.)

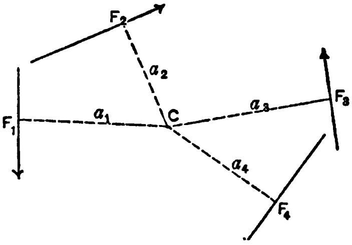

Fig. 27.
Equilibrium of Moments.

Let the forces F1, F2, F3 and F4, Fig. 27, act in a common plane and be in equilibrium. From any centre of rotation or point C let fall a perpendicular on the line of action of each force.

Then, $-F1\times a1+F2\times a2-F3\times a3+F4\times a4=0$.

This equation is called the "*Equation of Equilibrium of Moments.*" The moments of F1 and F3 are affected by the sign −, because each tends to rotate the plane in a negative direction; those of F2 and F4 by +, because positive. (61.) (103.)

Determination of an Unknown Force.

(105.) Let the forces be in a common plane and in equilibrium, but suppose that the magnitude of F2, for example, is unknown, though both its line of action and lever-arm are given.

Then $-F1\times a1+F2\times a2-F3\times a3+F4\times a4=0$, as before.

Transposing, $+F2\times a2=+F1\times a1+F3\times a3-F4\times a4$.

Reducing, $F2=\dfrac{F1\times a1+F3\times a3-F4\times a4}{a2}$

All the quantities on the right hand side of the equation being known, the numerical magnitude of F2 can easily be computed.

This simple principle forms the basis of Ritter's method.

General Principles.

(106.) 1. If any number of forces are in equilibrium, and act at a common point or in a common plane, the sum of their moments = 0.

2. The moment of any force whose line of action passes through the centre of rotation, equals 0.

3. The external loads and forces acting at any joint of a truss are always in equilibrium with the strains in the members, which meet at that joint, or the joint would move.

4. Each joint then being in equilibrium, the entire truss, or any portion of it, must be so likewise.

5. Consequently, any portion of the truss may be cut off from the remainder and separately considered, without destroying its equilibrium.

6. The line of section dividing the truss may be straight or curved, but must not cut more than three members whose strains are unknown.

7. The sense or direction of the strain acting in any cut member is always assumed to be directed towards the line of section from the part of the truss considered. (Figs. 29, 31.)

8. The centre of rotation, for determining the magnitude of the strain in any cut member is always to be taken at the intersection of the other two cut members, since the moment of the strains in these last members then equal 0, and may be omitted from the equation of moments, because both their lines of action pass through the centre of rotation.

9. The sign + prefixed to the numerical magnitude of a strain indicates that this strain is tension; —, that it is compression.

A clear understanding of these principles will be best gained by carefully examining their application to an example, with subsequent practice.

A few authors—Dubois, for example—employ — to indicate tension, and + for compression, but the method here given is the one more commonly employed.

APPLICATION TO A ROOF TRUSS.

Programme of conditions.

(107.) Let the truss be of the form shown in Fig. 28, having a span of 40 feet, with the trusses placed 16 feet apart. Rise of upper chord, 12 feet; of lower one, 2 feet.

Then $\frac{2 \text{ rise}}{\text{span}} = \frac{2\times12}{40} = .600 =$ tan. 31 degrees nearly, which is the angle of inclination of the roof. (This angle may also be directly measured with a protractor.)

The roof is to be covered with tin, laid on inch pine sheathing, which rests on 2×8 rafters, set 18 inches between centres. These are supported by 8×12 purlines, which rest on the trusses, at the loaded points B, D and F.

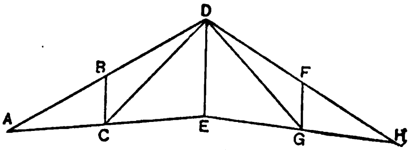

Fig. 28.
Diagram of Truss.

The maximum snow load is assumed to be 20 lbs. on a horizontal surface; maximum wind pressure 50 lbs. on a vertical surface; both taken per square foot. (Chap. III.)

By the table for wind pressures (101), we find that the wind pressure perpendicular to a surface having an inclination of 31 deg. is 34 lbs. per square foot.

Length of principal, $AD = \sqrt{20^2\times12^2} = 22.32$ feet; of AB or $BD = 11.66$ feet.

Therefore, a section area $= 11.66\times16 = 186.6$ square feet; = inclined area of roof actually supported at B, D or F.

The horizontal projection or horizontal area covered by this section area $= 16\times10 = 160$ square feet, since the truss is divided in 4 panels of equal length.

Computations of loads on truss.

(108.) Tin,	1 lb. per square foot.			
Sheathing,	3 lbs	"	"	
Rafters,	$2\frac{2}{3}$	"	"	
Total,	$6\frac{2}{3}$ lbs.	'	"	of roof.

Permanent load per section area:

Tin, sheathing and rafters $= 186.6\times6.6\frac{2}{3} = 1{,}244$ lbs.

Purline, 8×12, 3 lbs. per foot B. M. $= 384$ lbs.

Truss $= 160\times2.\ 1$ lb. $= 336$ lbs.

Total permanent load per section area and loaded point, 1,964 lbs.

Snow, per section area $= 160\times20 = 3{,}200$.

Total permanent and snow load supported at each ot the loaded points, B, D and F = 5,164 lbs. = 2.582 tons.

Since there are three loaded points upon the upper chord, the entire permanent and snow load supported by the truss = 2.582×3 = 7.746 tons. (The half loads at A and H are not included, because they are directly supported by the walls.)

Wind pressure per section area = 186.6×34 = 6,344 lbs. = 3.172 tons.

Total permanent load supported by truss = .982×3 = 2.946 tons.

Total wind pressure supported by truss = 3.172×1.5 = 4.758.

If the wind acts on the left-hand side of the roof, a full wind load is supported at B, and a half load at D; none at F.

The maximum snow load and wind pressure can hardly be found on the same side of the roof at the same time. The maximum strains in the members of the truss will most probably be found by first considering the truss as supporting the permanent and maximum snow loads; afterwards, the permanent load and maximum wind pressure.

Strains caused by permanent and snow loads.

(108*a*.) The total load on the truss is then 7.746 tons, equally divided between the points B, D and F. Hence, one-half this, or 3.873 tons, is supported at A and also at H, causing an equal upward reaction in each wall, which acts as an upward force, so as to make the truss in equilibrium. (Fig. 29.)

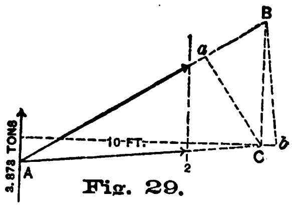

Fig. 29.

Strain in AB, centre of rotation at C.

Cut off part of the truss by a line 1-2, removing the remainder, as in Fig. 29, and take centre of rotation anywhere on AE, as at C, so as to reduce moment of strain in AC to 0. (106–8.)

Let fall perpendicular C*a* on AB and measure its length, which is found to be 4.28 feet, and is the lever-arm of the strain in AB; the lever-arm of the reaction at A = 10 feet.

Let (AB) represent the expression "*strain in the member* AB."*

---

* First employed by Prof. W. H. Burr.

The equation of equilibrium of moments will then be:

$+(AB\times 4.28+3.873\times 10=0.$ (104, 105, 106.)

Transposing and reducing:

$+(AB=-\frac{3.873\times 10}{4.28}=-9.049$ tons, = the compression acting in member AB. (106–9.)

The direction of the required strain being always taken *towards* the line of section, the moment of (AB) must be positive or +.

Strain in AC, centre of rotation at B. Fig. 29.

Let fall the perpendicular B*b* on AC, = 4.96 feet = lever-arm of (AC); that of reaction at A = 10 feet.

$-(AC)\times 4.96+3.873\times 10=0.$

Transposing, reducing and changing signs to make (AC)+.

$+(AC)=+\frac{3.873\times 10}{4.96}=+7.809$ tons = tension in AC.

Strain in BC, centre of rotation at A. Fig. 30.

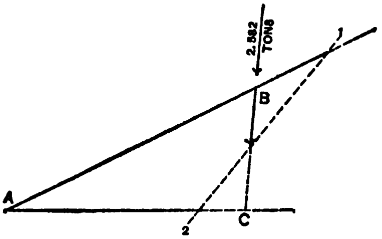

Fig. 30.

The moments of the strains in AC and AD, and of the reaction at A, each = 0, leaving only the load of 2.582 tons at B. (106–8.)

$+(BC)\times 10+2.582\times 10=0.$

$+(BC)=-\frac{2.582\times 10}{10}=-2.582$ tons compression = (BC).

Strain in DC, centre of rotation at A. Fig. 31.

Draw line of section 1–2; produce DC and let fall on it the perpendicular A*a* = 6.72 feet, = lever-arm of CD.

$-(CD)\times 6.72+2.582\times 10=0.$

$+(CD)=+\frac{2.582\times 10}{6.72}=3.842$ tons tension.

Strain in BD, centre of rotation at C. Fig. 31.

$$+(\mathrm{BD})\times 4.28+3.873\times 10=0.$$

$$+(\mathrm{BD})=-\frac{3.873\times 10}{4.28}=-9.049 \text{ tons compression.}$$

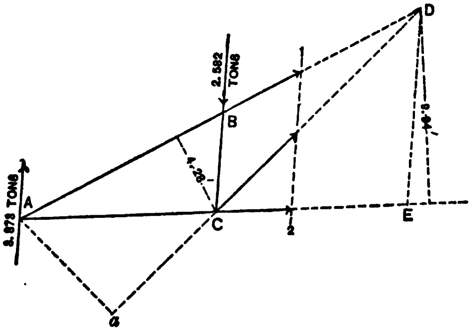

Fig. 31.

Strain in CE, centre of rotation at D. Fig. 31.

$$-(\mathrm{CE})\times 9.94+3.873\times 20-2.582\times 10=0.$$

$$+(\mathrm{CE})=+\frac{3.873\times 20-2.582\times 10}{9.94}=+5.195 \text{ tons tension.}$$

Strain in DE, centre of rotation at H. Fig. 32.

Draw the curved line of section 1–2–3; let fall the perpendicular Ha on CE produced; Ha = 3.98 feet = lever-arm of the strain in CE; lever-arm of DE = 20 feet.

$$+(\mathrm{DE})\times 20-(\mathrm{CE})\times 3.08=0.$$

Since (CE) has just been found = + 5.195 tons;

$$+(\mathrm{DE})=\times\frac{5.195\times 3.98}{20}=+1.034 \text{ tons tension.}$$

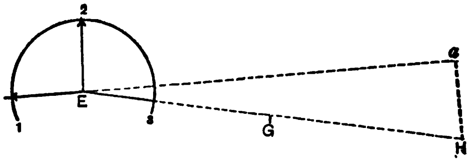

Fig. 32.

The truss being uniformly loaded, when supporting the maximum permanent and snow loads, the strains acting in the members of the right half of the truss will be the same as those already found in the corresponding members of the left hand.

Strains caused by permanent and wind loads.

(110.) Total permanent loads on truss = 2.046 tons.

Total wind loads on truss = 4.758 tons.

Since the former act vertically, and the latter are perpendicular to left side of the truss, assuming the wind to act on the left hand side of the roof, their resultant must be inclined. Its value may be easily found graphically.

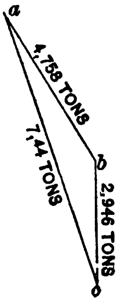

Fig. 33.

In Fig. 33, make *bc* vertical and *ab* perpendicular to left side of truss, respectively equal to 2.946 and 4.758 tons, at any convenient scale. Join *ac*, which will measure 7.44 tons, and which is the required resultant of all loads on the truss, and also equals the sum of the reactions at A and H, which must be parallel to *ac*, the truss being fixed to each wall.

(111.) In Fig. 34, draw the truss diagram (81), also reactions at A and H, parallel to *ac* of Fig. 33. At B and D make

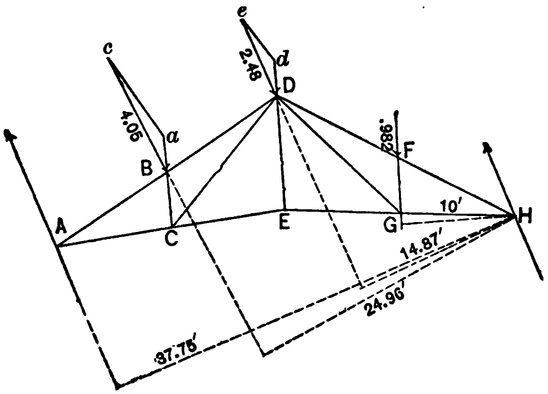

Fig. 34.

verticals B*a* and D*d* each = permanent load of 0.982 tons, at any convenient scale; also, draw *ac* and *de* perpendicular to AD, making *ac* = one wind load = 3.172 tons, and *de* = a half wind load = 1.586 tons; join B*c* and D*e*, which will represent the resultants of permanent and wind loads, acting at B and D; B*c* = 4.05, and D*e* = 2.48 tons. At F is a vertical permanent load of 0.982 tons.

Produce lines of action of the reaction at A and of the resultants at B, D and F, and let fall on each a perpendicular from H, which will be the lever-arm of each force for H as a centre of rotation, and measure lengths of these lever-arms. That of reaction at A = 37.75 feet, those of resultants at B, D and F, are 24.96, 14.87, and 10 feet as given in the figure.

(112.) Hence, for *Reaction at A, centre of rotation at H.* + (Reaction at A)×37.75−4.05×24.96−2.48×14.87−.982×10 = 0.

$$\text{Reaction at A} = \frac{4.05\times24.96+2.48\times14.87+.982\times10}{37.75} = 3.015$$ tons.

The reaction at H therefore = 7.44−3.915 = 3.525 tons.

(113.) To determine the strains on the different members, the lever-arm of each force or member is to be found as before, by letting fall a perpendicular from the centre of rotation on the corresponding line of action or member, then measuring the length of this perpendicular, which will be the required lever-arm for that force or member. This is shown in Figs. 35 and 36, but it is not thought necessary to illustrate the process of obtaining the strain in each member by separate figures as before. (109.)

(114.) For the left side of the truss, the equations are as follows:

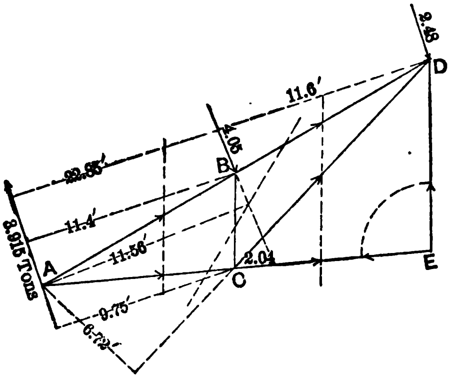

Fig. 35.

Strain in AB, centre of rotation at C. (Fig. 35.)
+(AB)×4.28+3.915×9.75 = 0.

$$(AB) = \frac{3.915\times9.75}{4.28} = -8.918 \text{ tons compression.}$$

Strain in AC, centre of rotation at B. (Fig. 35.)

$$-(AC)\times 4.96+3.915\times 11.40=0.$$

$$(AC)=+\frac{3.915\times 11.40}{4.96}=+8.998 \text{ tons tension.}$$

Strain in BC, centre of rotation at A.

$$+(BC)\times 10+4.05\times 11.56=0.$$

$$(BC)=-\frac{4.05\times 11.56}{10}=-4.682 \text{ tons compression.}$$

Strain in CD, centre of rotation at A.

$$-(CD)\times 6.73+4.05\times 11.56=0.$$

$$(CD)=+\frac{4.05\times 11.56}{6.72}=+6.96 \text{ tons tension.}$$

Strain in BD, centre of rotation at C.

$$+(BD)\times 4.28+3.915\times 9.75+4.05\times 2.04=0.$$

$$(BD)=-\frac{3.915\times 9.75+4.05\times 2.04}{4.28}=-6.988 \text{ tons compression.}$$

Strain in CE, centre of rotation at D.

$$-(CE)\times 9.94+3.915\times 22.85-4.05\times 11.60=0.$$

$$(CE)=+\frac{3.915\times 22.85-4.05\times 11.60}{9.94}=+4.273 \text{ tons tension.}$$

Strain in DE, centre of rotation at H.

$$+(DE)\times 20-(CE)\times 3.98=0.$$

$$(DE)=+\frac{4.273\times 3.98}{20}=+0.869 \text{ tons tension.}$$

(115.) For the right half of the truss, commence at H and proceed toward the middle of truss. The strains in the members of the right half must be obtained, as they differ from those already found in the corresponding members of the left half.

The equations are as follows:

Strain in HF, centre of rotation at C. (Fig. 36.)

$$-(HF)\times 4.28-3.525\times 9.12=0.$$

$$(HF)=-\frac{3.525\times 9.12}{4.28}=-7.511 \text{ tons compression.}$$

Strain in HG, centre of rotation at F. (Fig. 36.)

$$+(HG)\times 4.96-3.525\times 7.48=0.$$

$$(HG)=\frac{3.525\times 7.48}{4.96}=+5.316 \text{ tons tension.}$$

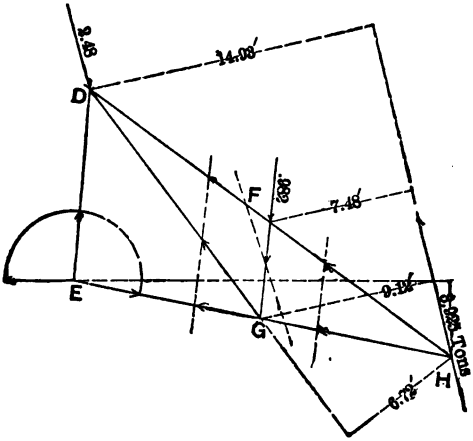

Fig. 36.

Strain in FG, centre of rotation at H.

$-(FG)\times10-.982\times10=0.$

$(FG)=-\frac{.982\times10}{10}=-0.82$ tons compression.

Strain in DG, centre of rotation at H.

$+(DG)\times6.72-0.982\times10=0.$

$(DG)=+\frac{0.982\times10}{6.72}=+1.461$ tons tension.

Strain in DF, centre of rotation at G.

$-(DF)\times.428-3.525\times9.12=0.$

$(DF)=-\frac{3.525\times9.12}{4.28}=-7.511$ tons compression.

Strain in GE, centre of rotation at D.

$+(GE)\times9.94-3.525\times14.93-0.982\times10=0.$

$\frac{(GE)=+3.525\times14.93-0.982\times10}{9.94}=+4.307$ tons tension.

The Strain Sheet.

(116.) After determining the strains acting in each member of the truss for permanent and snow and permanent and wind loads, the results must be collected in a table or strain sheet, like that here given, to more conveniently determine the maximum strain which may act on each member, and whether any mode of loading may reverse the strain, that is, cause tension in a member usually subject to compression, etc. If this be the case, such member must be so designed as to safely resist *both* the maximum compression and tension which may act on it (86), (89). It will be sufficient to write out the strain sheet for the left half of the truss only, as the truss is symmetrical, and the strains found in the members of the right side, when the wind acts on the left side of the roof, are identical with those in corresponding members of left side, with the wind acting on the right side.

In the present example no strains are reversed, each member being subject to compression or tension only.

So far as known to the writer, Ritter's Method has previously been applied only to the vertical components of the wind forces, neglecting their horizontal components, thus introducing a serious error. It is evidently as readily applicable to the actual inclined forces, since their lines of action are easily found, and the lengths of the lever-arms may then be found by measurement on the truss diagram, as here explained.

MEMBER.	P. & S.	P. & W. WINDWARD.	P. & W. LEEWARD.	MAXIMUM.
AB..........	−9.049	− 8.918	−7.511	− 9.049
BD..........	−9.049	−10.848	−7.511	−10.848
AC..........	+7.809	+ 8.998	+5 316	+ 8.998
CE..........	+5.195	+ 4.273	+4.307	+ 5.195
BC..........	−2.582	− 4.682	−0.982	− 4.682
CD..........	+3.842	+ 6.967	+1.461	+ 6.967
DE..........	+1.034	+ 0.869	+0.869	+ 1.034

Compression is denoted by — ; tension by +.

The maximum strains in AB, CE and DE are caused by the P. & S. loads; the maximum strains in running numbers by P. & W. loads.

CHAPTER V.

THE GRAPHICAL METHOD.

This method will here be applied to several forms of trusses, so selected as to comprise most of the difficulties found in practice; it can easily be applied to any given form of truss by the reader, if the given examples are carefully studied.

Programme of Conditions for Problem 1.

(117.) Truss to be of form shown in Fig. 37; span 80 feet; rise of upper chord 15 feet; trusses placed 16 feet apart between centres, each being divided in 8 equal panels.

Roof covered with tin, laid on inch pine sheathing, supported by 2x6 rafters, spaced 24 inches between centres; rafters are supported by 8x10 purlines set edgewise, one to each panel.

Maximum snow load (for New York City, Chicago, etc.) = 20 lbs. per square foot of horizontal area covered by the roof; maximum wind pressure = 50 lbs. per square foot on a vertical plane surface; these loads are the same for all the trusses treated in this chapter.

Length of principal rafter $= \sqrt{40^2+15^2} = 42.72$ feet; panel length of upper chord $= 42.72 \div 4 = 10.68$ feet; section area of roof $= 10.68 \times 16 = 171$ square feet nearly.

Inclination of surface of roof $= 20\frac{1}{2}$ degrees nearly (107). The maximum wind pressure normal to roof surface will then be $= (23.0+24.1) \div 2 = 23.6$ lbs. nearly.

Computation of Loads Supported by one Truss.

(118.)

Tin and paint,	1	lb. per square foot.	
Sheathing,	3	"	"
Rafters,	1.5	"	"
Total,	5.5	"	"

Load per section area or point of upper chord.
Tin, sheathing and rafters = 171×5.5 = 941 lbs.
Purline, 8×10 = 106⅔ feet B. M.×3 = 420 lbs.
Truss = 160×4¼ lbs. = 680 lbs.
Total permanent load = 2041 lbs. = 1.021 tons.
Snow load = 160×20 = 3200 lbs. =1.600 tons.
Wind load = 171×23.6 = 4036 lbs. = 2.018 tons.

Since the truss is divided in 8 equal panels, and one-half of each end panel rests directly on the walls, the truss evidently supports 7 section areas of the roof.

Hence, total permanent and snow load supported by one truss = 7 (1.021+1.600) = 18.347 tons.

(119.) *Notation Employed.*

Bow's notation, modified, is the best and most simple. See Fig. 37.

Call the entire surface of the paper above the truss diagram, X; that below it, Y; then number each triangle, composing the truss, in regular order from left to right; name any member of the truss, from the letter and number, or the two numbers denoting the two surfaces separated by that member.

Letter the ends of the line in the strain diagrams, which represents the strain acting in this member, the same as the surfaces separated by the member. Compare Figs. 37, 38, 39, etc.

After this system of notation is clearly understood, it will be found to materially aid in drawing the strain diagrams of a complex form of truss.

(120.) *Strain Diagram for Permanent and Snow Loads.*

Draw the truss diagram to any convenient scale, as in Fig. 37, as large as possible.

Taking any convenient scale of tons to the inch, draw a vertical line in Fig. 38, making its length = 18.347 tons = total permanent and snow load on truss. Divide this load line into 7 equal parts, each part representing the load at B, D, F, etc.; bisect the load line at Z, and its lower half = that part of the truss and its load supported by the right hand wall, the upper half = load on left hand wall, each being = 9.174 tons nearly.

Since the end of the truss cannot move, each wall must exert an upward pressure exactly equal to the downward pressure of the truss on it, and this upward pressure is represented by the same half of the load line. The upward pressures or *reactions* of the walls are always considered instead of the downward pressure, because they hold the truss in equilibrium with the loads acting upon it.

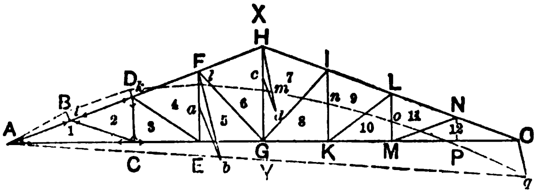

Fig. 37.
Triangular Truss.

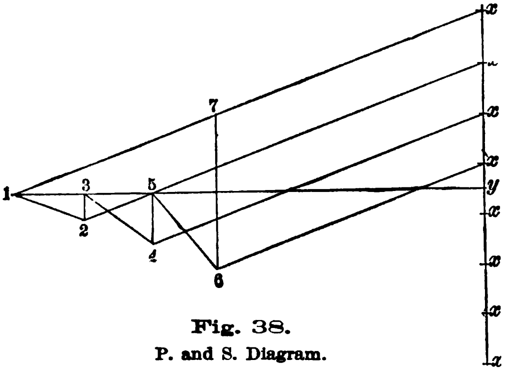

Fig. 38.
P. and S. Diagram.

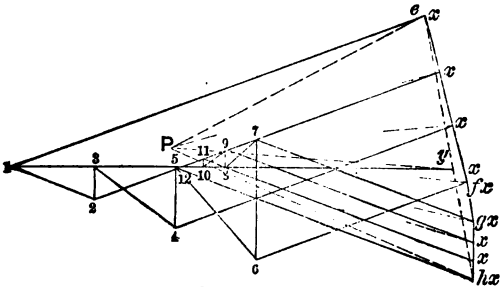

Fig. 39.

Therefore, we have a vertical force of 9.174 tons acting upwards at A, and represented by the upper half of load line, Fig. 38, and which must be in equilibrium with the strains acting in the two members X1 and Y1, which meet at A. (106.)

Draw in Fig. 38, *x*1 and *y*1 parallel to X1 and Y1 of Fig. 37, intersecting at 1; these lines will represent the strains acting in the corresponding numbers X1 and Y1, and their magnitudes in tons may be measured by applying the same scale used in laying off the load line.

At B, Fig. 37, we have the load, represented by the upper division *xx* of the load line, and the strain in X1, represented by *x*1, to find the unknown strains acting in X2 and 1 2.

Draw *x*2 parallel to X2 and through 1; draw 1 2 parallel to 1 2 of Fig. 21, intersecting at 2. Then *x*2 and 1 2 represent the required strains in X2 and 1 2 as before.

No load acts at C, but the strains acting in Y1 and 1 2 are known, while those acting in *y*3 and 2 3 are required. Through 2 draw 2 3 parallel to 2 3 of Fig. 37, intersecting *y*1, with which *y*3 must coincide, at 3; *y*3 and 2 3 represent the required strains in *y*3 and 2 3.

The strain diagram for the left half of the truss is completed by taking the joints in such order that not more than two unknown strains are found at any joint. The strain diagram for the right half of the truss is best drawn by commencing at the end O, then proceeding towards middle of the truss; it is merely a duplicate of that for the left half, and is omitted in Fig. 38 for the sake of clearness.

(121.) *Resultants of Permanent and Wind Loads.*

The wind is assumed to act on the left side of the roof, so that full wind loads are supported at B, D and F, with a half wind load at H. Since the permanent load acts vertically and the wind load is normal to the roof, their resultant must be found graphically, then substituted for them.

With any convenient scale as large as possible in Fig. 37, make the vertical F*a* = 1.021 tons = permanent load; make *ab* perpendicular to AH and = 2.018 tons = wind load; join F*b*, which measures 3.00 tons by the same scale = required resultant at F and also at B and D. Through B and D draw lines parallel to F*b*. Make H*c* = 1.021 tons, and *cd* perpendicular to AH and = 1.009 tons = half a wind load; H*d* measures 1.99 tons = resultant at H. Each resultant is evidently considerably less than the sum of a permanent and wind load.

(122.) *Strain Diagram for Permanent and Wind Loads.*

There are equal resultants at B, D and F, a smaller one at H, with equal permanent loads only at I, L. & N.

With the same scale used in Fig. 38, draw *ef* in Fig. 39 parallel to F*b* and $=3.00\times3=9.00$ tons; draw *fg* parallel to H*d* and $=1.99$ tons; also, make *gh* vertical and $=1.021\times3=3.063$ tons; divide *ef* and *gh* into three equal parts. The loads on the truss, from left to right, will then be represented in order, from *e* to *h* on the load line. Join *eh*, which equals and is parallel to the resultant of all the forces acting on the truss; consequently, the sum of the reactions (or upward pressures of the walls) at A and O is represented by *eh*, to which they must be parallel, as no expansion rollers are here used.

The magnitude of each reaction must be found before the strain diagram can be drawn, and this is most readily done by the method of the Equilibrium Polygon (Fig. 33).

Select any pole P, and draw lines connecting it with the divisions of the load line (Fig. 39). Commencing at A in Fig. 37, draw the inverted equilibrium polygon A *iklmnopq*, its sides parallel to the strings in (Fig. 39), taken in order downwards, and intersecting on the lines of action of the resultant forces, acting at the joints of the truss; the last side P*q* intersects at *q*, *a* parallel to *eh* (Fig. 39), drawn through O (Fig. 37). Join A*q*, and parallel to A*q*. Fig. 37, draw P*y* in Fig. 39, cutting *eh* at *y*. Then *ey* represents the reaction at A, *yh* that at O.

The strain diagram is then drawn in the same way as that for the permanent and snow loads, commencing at A to draw the left-hand side, and at O for the right-hand side. The magnitudes of the strains are then measured as before.

(123.) *Checks on the Accuracy of the Work.*

1. The strain diagram must close, *i. e.*, the middle line 6 7 of Fig. 38 or 39, and which is drawn last, must be parallel to the corresponding member 6 7 of Fig. 37. True in all cases.

2. An *x*-line, a *y* line, and each alternate vertical, intersect at a common point. True for most triangular trusses.

3. Apply Ritter's Method (Chap. IV.) to find the strains in a few members, which must be sensibly equal to the values obtained graphically.

(124.) *Determination of Nature of Strain.*

Let a load W be supported by a post AB, as in Fig. 40. The load evidently acts at A as a downward force, and is necessarily resisted by an equal upward force acting within the post at A. Neglecting the weight of the post, an equal downward force must act within the post at B.

Hence, if the internal forces acting within a member are found to act from its middle towards its ends, as in Fig. 40, the member must be subject to compression.

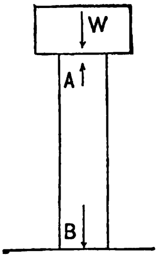

Fig. 40—Compression.

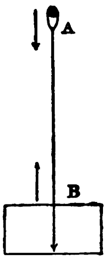

Fig. 41—Tension.

Let a load W be supported by a rope AB, as in Fig. 41. The load acts at B in a downward direction, and is resisted by an equal upward force within the rope, producing an equal downward force at A.

Therefore, if the internal forces act from the ends of a member towards its centre, as in Fig. 41, this member must be subject to tension.

Take joint A, Fig. 37. The corresponding force polygon in Fig. 38 is xy 1, because the sides of this triangle represent the three forces acting at A. We know that these three forces must be in equilibrium at A, and that the line xy represents the reaction of the wall, which must act upwards; consequently at A, the strain in Y1 must act in the direction from 1 towards y, and that in X1 must act from x towards 1 in Fig. 38, because these three forces must act in the same direction around the triangle or force polygon xy 1, as they are in equilibrium at A. (22.) Hence X1 is subject to compression and Y1 to tension, since in the former the internal strain acts from the middle towards the ends of the member X1, and *vice versa* in Y1

Take joint B, Fig. 37. The corresponding force polygon in Fig. 38 must be the polygon xx 2 1, the load at B being represented by xx. This load acts downwards, therefore the strain in X1 must be in the direction from 1 to x, Fig. 38; the strain in 1 2, from 2 towards 1; the strain in X2, from x towards 2; all the members meeting at B are therefore in compression.

Take joint C. The force polygon will be $y3$ 2 1, $y3$ and $y1$ coinciding, because both are drawn through the same point y, parallel to the same straight line AO. Since the strain in Y1 acts from left to right at A, it must act from right to left at C, because the strains at the ends of any member must always be *equal* in magnitude, but *opposed* in direction; otherwise the member would be moved endwise in the direction of the greater strain, or in that of both strains, if they had the same sense, and this member could not then be in equilibrium, as required. Consequently, at C, the strain in Y1 must act from y towards 2 (Fig. 38); that in Y3 acts from 3 towards 2; that in 2 3, from 2 towards 3; that in 1 2, from 1 towards 2. Then Y1, Y3 and 2 3 are evidently subject to tension, and 1 2 to compression, as already found at B.

The same kind of strain being found to act in 1 2 at both its ends B and C, proves that the nature of the strains in all the members meeting at C have been correctly determined.

The kinds of strain acting in the remaining members of the truss may be found by continuing the use of the method just explained, which must also be applied to the strain diagram for permanent and wind loads (Fig. 39), because the nature of the strain in a member is sometimes changed by the wind pressure, especially in curved roofs. (Prob. 4.) Great care must be taken to obtain a clear knowledge of this method by applying it practically.

(125.) *The Strain Sheet.*

Three different intensities of strain are found to act on each member of the truss, caused by the permanent and snow loads, and by the permanent and wind loads; in the last case, different strains are found on the windward and leeward sides of the roof. The line x 12 (Fig. 39) represents the strain acting in X1, for example, when the wind acts on the right-hand side of the roof, etc.

After measuring the magnitudes of these strains with the same scale used in laying off the load lines (Figs. 38 and 39), and determining their nature, collect them in a table as follows, indicating tension by + and compression by —. (106.)

STRAIN SHEET FOR PROBLEM 1.

MEMBER.	P. & S.	WINDWARD. P. & W. W.	LEEWARD. P. & W. L.	MAXIMUM.
	Tons.	Tons.	Tons.	Tons.
X 1.........	−26.10	−22.90	−16.42	−26.10
X 2.........	−22.32	−19.10	−14.90	−22.32
X 4..... ...	−18.64	−15.30	−13.46	−18.64
X 6.........	−14.88	−11.49	−12.00	−14.88
Y 1.........	+24.42	+22.87	+14.36	+24.42
Y 3..... ...	+20.92	+18.63	+12.93	+20.92
Y 5.........	+17.42	+14.37	+11.58	+17.42
1 2.........	− 3.77	− 4.56	− 1.54	− 4.56
3 4.........	− 4.37	− 5.84	− 1.70	− 5.84
5 6.........	− 5.26	− 6.40	− 2.07	− 6.40
2 3.........	+ 1.32	+ 1.62	+ 0.55	+ 1.62
4 5.........	+ 2.63	+ 3.17	+ 1.05	+ 3.17
6 7	+ 7.87	+ 6.30	+ 6.30	+ 7.87

The maximum strains in the upper and lower chords are evidently produced by the permanent and snow loads, but those in the web members by permanent and wind loads, with the sole exception of that in the vertical 6 7.

(126.) *Programme of Conditions for Problem* 2.

The type of truss, its dimensions, loads, etc., to be exactly as in Problem 1, with the following exceptions:

1. An equal portion of the weight of the truss is assumed to be supported at each joint of the upper and lower chords.

2. The truss is also required to support a lathed and plastered ceiling, attached to joists, their ends being supported by the horizontal tie-beam of the truss. A rough floor is laid on these joists, but no load is to be placed on it.

(127.) *Computation of Loads.*

Weight of truss $= 80 \times 16 \times 4.25 = 5440$ lbs. As there are 16 joints in the truss, $5440 \div 16 = 340$ lbs. $=$ load at each joint.

Permanent load per joint of upper chord:

Tin, sheathing and rafters (118).	941 lbs.
Purline	320 "
Truss	340 "
Total	1601 lbs.

Permanent load, 1601 lbs. =	.801 tons.
Snow load (118), 3200 lbs. =	1.600 "
Total permanent and snow load =	2.401 tons.

Total permanent and snow load on upper chord = 2.401 × 7 = 16.807 tons.

Wind load per section area as before (118), = 2.018 tons.

Permanent load per joint of lower chord. The ceiling joists are 2 × 8, 16 inch centres, with a rough floor laid on top of joists, but without any load on it.

Lathing and plastering, 3-coat work	10 lbs.
Rough floor	3 "
Joists	4 "
Total per square foot	17 lbs

Ceiling = 160 × 17 =	2720 lbs.
Truss, as before	340 "
Total per joint	3060 lbs. = 1.53 ton.

Total permanent load for lower chord = 1.53 × 7 = 10.71 tons.

(128.) *Strain Diagram for Permanent and Snow Loads.*

Draw the truss diagram (Fig. 42) to the same scale as that of Fig. 37.

With the scale used in Fig. 38, draw the vertical load line de in Fig. 43, making it = 16.807 tons; divide it in 7 equal parts, marking the points of division x, also bisecting it at a. On the same vertical line, lay off ab and ac, each = 10.71 ÷ 2 tons, and divide bc into 7 equal parts, marking these points of division y.

The load line de represents the total load on the upper chord, one part ax also representing the load at one joint; bc represents

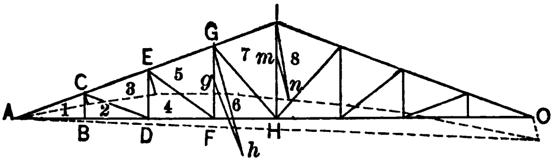

Fig. 42
Triangular Truss.

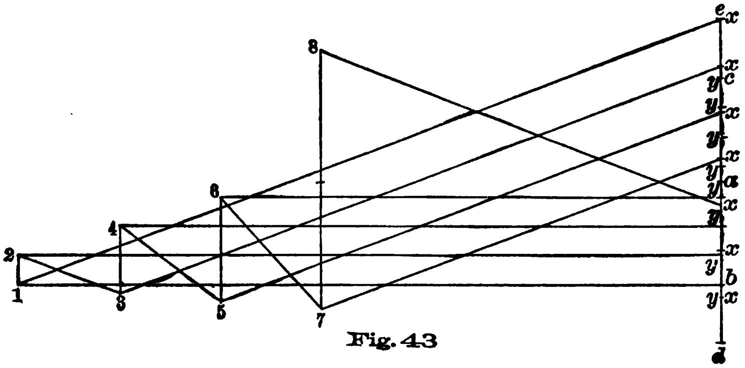

Fig. 43
P. and S. Diagram.

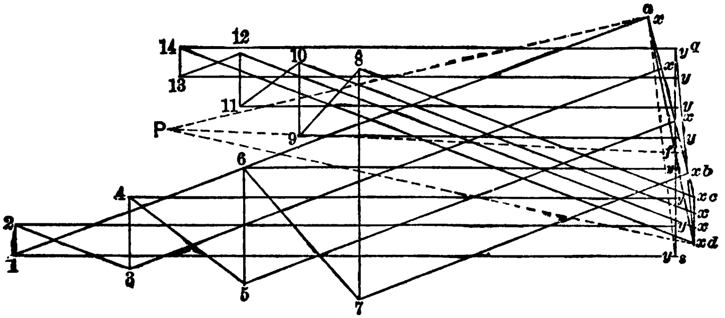

Fig. 44
P. and W. Diagram.

the total load on the lower chord, one part *yy* being the load at one joint.

Joint A. The reaction evidently = half the total load on the entire truss, = *ac* + *ab* = *bc*. Through *b*, draw *y*1 parallel to Y1 of Fig. 42, and through *c*, draw *x*1 parallel to X1; these intersect at 1, and *y*1 represents the strain in Y1, and *x*1, that in X1 of the given truss.

Joint B. Draw 1 2 and *y*2 parallel to 1 2 and Y2 of Fig. 42, intersecting at 2.

Joint C. Draw 2 3 and *x*3 parallel to the corresponding members of the truss, intersecting at 3.

The mode of completing the strain diagram is sufficiently obvious. For sake of clearness, only one-half the diagram is here given.

(129.) *Strain Diagram for Permanent and Wind Loads.*

In Fig. 42, make *g*G and I*m* each = .801 ton; also, *gh* = a wind load = 2.018 tons, and *mn* half a wind load, = 1.009 tons; join G*h*, which measures 2.78 tons, and I*n*, which = 1.78 tons.

In Fig. 44, make *ab* parallel to G*h* of Fig 42, and = 2.78 × 3 = 8.34 tons; *bc* parallel to I*n* and = 1.78 tons; *cd* vertical and = .801 × 3 = 2.403 tons; using the same scale as in Fig. 43. Join *ad*, which will be the resultant of the permanent and wind loads on the upper chord only.

Choose a pole P, and by the method of the Equilibrium Polygon, as applied to Problem 1, find the dividing point *f*; then *af* = reaction at A, and *fd* = reaction at O, for the loads on the upper chord only.

Through *f* draw a vertical line, and make *fq* and *fs* each = *ab* or *ac* of Fig. 43 = 10.71 ÷ 2 tons; divide *qs* in 7 equal parts. Then *afs* or the resultant *as* = total reaction at A, and *qfd* or the resultant *qd* = total reaction at O.

Joint A. Through *a* and *s* (Fig. 44) draw *x*1 and *y*1 parallel to X1 and Y1 of Fig. 42, intersecting at 1.

The remainder of the diagram or the left half of the truss is then completed as in Fig. 43.

For the right-hand half, commence at O, drawing through *d* and *q* of Fig. 44, *x*1 and *y*1 parallel to X14 and Y14 of Fig. 42, etc.

The y-strain lines must be measured from the vertical line qfs ; the x-strain lines, from the broken load line $abcd$.

(130.) *The Strain Sheet.*

The strains on the different members are then measured, and collected in the following table, as in the preceding case.

STRAIN SHEET FOR PROBLEM 2.

MEMBER.	P. & S.	P. & W. W.	P. & W. L.	MAXIMUM.
X 1.........	−39.08	−35.24	−28.60	−39.08
X 3	−33.40	−29.65	−25.30	−33.40
X 5.........	−27.82	−24.02	−21.97	−27.82
X 7.........	−22.22	−18.38	−18.68	−22.22
Y 1.........	+36.58	+34.42	+25.85	+36.58
Y 2.........	+36.58	+34.42	+25.85	+36.58
Y 4.........	+31.28	+28.48	+22.74	+31.28
Y 6.........	+26.05	+22.52	+19.68	+26.05
1 2.........	+ 1.55	+ 1.55	+ 1.55	+ 1.55
3 4.........	+ 3.50	+ 3.75	+ 2.70	+ 3.75
5 6.........	+ 5.44	+ 6.05	+ 3.88	+ 6.05
7 8.........	+13.27	+11.77	+11.77	+13.27
2 3.........	− 5.66	− 6.35	− 3.34	− 6.35
4 5	− 6.52	− 7.46	− 3.86	− 7.46
6 7.........	− 7.90	− 9.03	− 4.74	− 9.03

(131.) Comparing this strain sheet with that of Problem 1 (125), we note the following points:

1. The strains in the members are increased about 50 per cent. on an average.

2. The maximum strains in the chords are still caused by P. and S. loads; in the web members, by P. and W. loads.

3. The Graphical Method is readily applicable in the manner indicated to a truss loaded in any manner at each joint, or at only a portion of the joints.

4. It can be applied to an iron truss supporting a ceiling, with expansion rollers at one end, by determining the y-points as shown hereafter in Problem 3, then drawing a vertical through each y-point, and laying off on each vertical the loads at the joints of the lower chord, as done from the point f in Fig. 44.

(132.) *Programme of Conditions for Problem* 3.

Truss to be of Fink type, as in Fig. 45; span, 80 feet; rise of upper chord, 17 feet; of lower chord, 2 feet; divided in 8 equal panels; trusses 16 feet apart between centres; snow load and wind pressure taken at 20 and 50 lbs. as in previous Problems.

Roof to be covered with Carnegie Bros.' corrugated iron, weighing 2 lbs. per square foot laid, without sheathing or common rafters; supported by two purlines to each panel or section

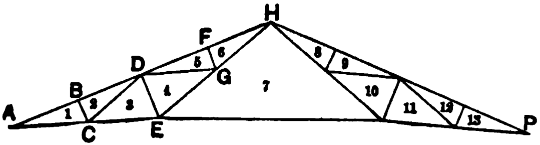

Fig. 45.
Fink Truss.

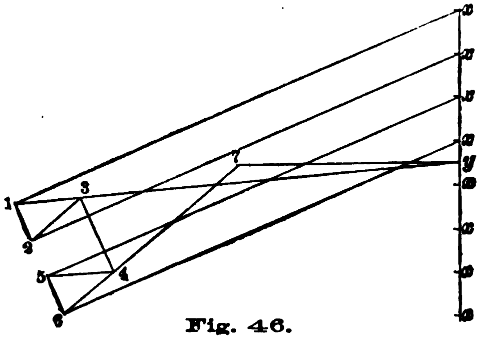

Fig. 46.
P. and S. Diagram.

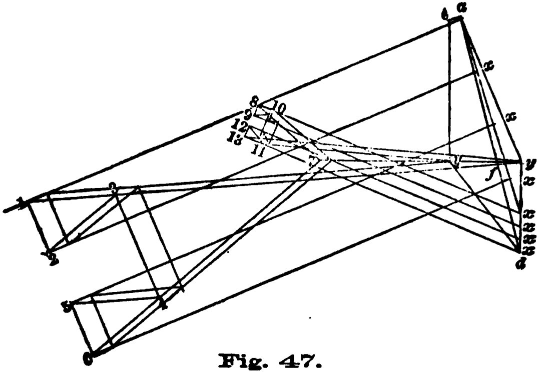

Fig. 47.
P. and W. Diagram.

area; each purline to be a 6 in. 10 lb. channel bar; roof truss to be of wrought-iron, with expansion rollers at one end, the other being fastened to the wall.

Length of principal $= \sqrt{17^2+40^2} = 43.46$ feet. Panel length of upper chord $= 43.46 \div 4 = 10.865$ feet. Section area $= 10.865 \times 16 = 174$ square feet, nearly. Half a section area $= 87$ square feet $=$ area of roof supported by one purline.

Inclination of roof $= 23$ degrees nearly; hence, maximum wind pressure $= 26.3$ lbs. per square foot. (101.)

(133.) *Computation of Loads on Truss.*

Corrugated iron $= 174 \times 2 =$. 348 lbs.
Purlines $= 2 \times 16 \times 10 =$ . . 320 "
Truss $= 160 \times 7.38 =$ . . . 1181 " (97.)

Total permanent load per section area $= 1849$ lbs $= .925$ ton.

Snow load $= 160 \times 20 = 3200$ lbs. $= 1.600$ tons.

Permanent and snow load $= .925 + 1.600 = 2.525$ tons.

Total permanent and snow load for the entire truss $= 2.525 \times 7 = 17.675$ tons.

Wind load per section area $= 174 \times 26.3 = 4577$ lbs. $= 2.289$ tons.

(134.) *Strain Diagram for Permanent and Snow Loads.*

The load line is laid off in Fig. 46 as in Problem 1 (120) $=$ 17.675 tons, and is then bisected at y, and also divided into 7 equal parts, corresponding to the number of loaded points on the truss. There is no ceiling, and the entire weight of the truss is assumed to be concentrated at the joints of the upper chord, as in Problem 1.

Commencing at A (Fig. 45), proceed as before until the member 3 4 is reached. Taking the joint D, we have three unknown strains in 3 4, $y4$ and 4 5; taking the joint E, we also have three unknown strains in 3 4, 4 7 and $y7$; the problem therefore becomes indeterminate. But the loads at B and F being equal, the strains in 1 2 and 5 6 must evidently be equal also; since the angles ACD and DGH are equal, and each angle is bisected by 1 2 or 5 6, equal strains will be caused in $y1$ and 2 3 by the pressure of the member 1 2, and in 4 5 and 6 7 by that of 5 6; therefore, the strains in 2 3 and 4 5 must be equal, but the magnitude of that acting in 2 3 has already been found, being represented by the line 2 3 of Fig. 46.

Consequently, the lines 3 4 and $x5$ of Fig. 46 must be connected by a line 4 5, parallel to 4 5 of Fig. 45, its length being equal to that of 2 3 of Fig. 46.

Draw 1 5 perpendicular to *x*1, and the point 5 thus found on *x*5 will be the only one through which the required line 4 5 can be drawn to satisfy the given conditions.

The strain diagram is now easily completed.

(135.) *Strain Diagram for Permanent and Wind Loads.*

Determine resultants at B, D, F and H, as in Problem 1, and lay off load line *abcd* as before; join *ad*, which would be the required resultant of all the loads acting on the truss, if the expansion rollers were omitted.

Divide *ad* at *f* into the reaction *af* acting at A, and *fd* acting at P, by the method of the equilibrium polygon.

1. *Rollers at A, or on Windward side of truss.* (38.)

The horizontal component of the loads on the lower half section area AB must be resisted by the truss, since it cannot be transmitted through the rollers to the wall at A. This is found equal to .45 ton by graphical construction. (32.) Make *ae* horizontal and equal to .45 ton, and let fall a vertical through *e* (Fig. 47), to intersect a horizontal drawn through *f*; their intersection will be the *y*-point, and *ey* will be the reaction at the left wall, *yd* that at the right-hand wall.

2. *Rollers at P, or at the Leeward.*

Draw a vertical through *d* to intersect a horizontal through *f*, and this point of intersection will be the *y*-point for rollers at leeward; *ay* will then be the reaction at the left, and *dy* at the right-hand wall.

It is evident that both these cases must be considered, because the wind may act on either side of the roof, and it is necessary to determine the maximum strains which may possibly occur.

The completion of the diagram offers no difficulty, and two strain diagrams are obtained, as in Fig. 47, having different *y*-points, but derived from the same load line, one corresponding to rollers at the windward, the other to rollers at the leeward side of the truss.

It is easier to shift the position of the rollers than the direction of the wind in the strain diagram, the same results being obtained.

(136.) *The Strain Sheet.*

The magnitudes of the strains are measured and their nature determined by the method already explained (124), thus obtaining five different strain values for each member, as shown in the accompanying table.

It is usual to take the larger strain as being the maximum for the member, without regard to the position of the rollers, because both sides of the truss are then made alike, economizing labor, though this involves the use of a slight excess of material on the side of the truss at which the rollers are placed.

STRAIN SHEET FOR PROBLEM 3.

MEMBER.	P. & S.	Rollers P. & W.W.	Windward P. & W. L.	Rollers P. & W. W.	Leeward P. & W. L.	Maximum
X 1..............	−28.60	−26.02	−17.38	−27.06	−18.37	−28.60
X 2..............	−27.60	−25.66	−17.04	−26.70	−18.02	−27.60
X 5..............	−26.60	−25.30	−16.66	−26.36	−17.67	−26.60
X 6..............	−25.60	−24.06	−16.30	−25.78	−17.30	−25.78
Y 1..............	+26.43	+23.60	+12.45	+28.17	+17.00	+28.17
Y 3..............	+22 66	+18 53	+11.10	+23.05	+15.60	+23.05
Y 7..............	+13.50	+ 7.45	+ 7.45	+11.57	+11.57	+13.50
1 2..............	− 2.30	− 3.10	− 0 85	− 3.10	− 0.85	− 3.10
3 4.......	− 4.65	− 6.30	− 1.70	− 6.30	− 1.70	− 6.30
5 6..............	− 2.30	− 3.10	− 0.85	− 3.10	− 0.85	− 3.10
2 3..............	+ 3.80	+ 5.05	+ 1.35	+ 5.05	+ 1.35	+ 5.05
4 5..............	+ 3.80	+ 5.05	+ 1.35	+ 5.05	+ 1.35	+ 5.05
4 7..............	+ 9.58	+11.30	+ 3.90	+11.90	+ 4.46	+11.90
6 7..............	+13.38	+16.38	+ 5.25	+16.97	+ 5.80	+16.97

(137.) *Programme of Conditions for Problem* 4.

Truss to be semicircular as in Fig. 48; depth of truss 10 feet at top; divided into 12 equal panels by radials; trusses 16 feet apart between centres; radials to be in tension and to be iron rods, if possible; diagonals to be in compression, and to be wooden timbers in any case; upper and lower chords of the truss to be built up of plank, bent to the curve and firmly fastened together.

Roof covered with tin, laid on inch sheathing of pine, which is supported by 4×8 pine purlines set edgewise and radially, and which are to be set 16 inches between centres at A, B and D, 18 inches at F, 24 inches at H, and 36 inches at K and M; common rafters omitted, as the sheathing rests directly on the purlines and is bent to the curve of the roof.

Length of upper chord $= 80 \times 3.1416 \div 2 = 125.67$ feet.

Panel length of upper chord $= 125.67 \div 12 = 10.47$ feet.

Section area = 10.47 × 16 = 167.5 square feet.

The inclination of a tangent at any joint of the upper chord may be taken as the average inclination of the section area of the roof, which is supported at that joint, and is most easily found by measurement with a protractor, after drawing the tangent and a horizontal line through each point.

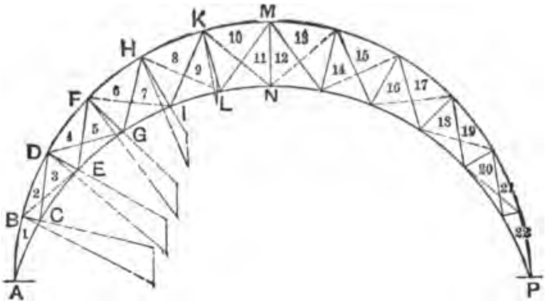

Fig. 48.
Crescent Truss.

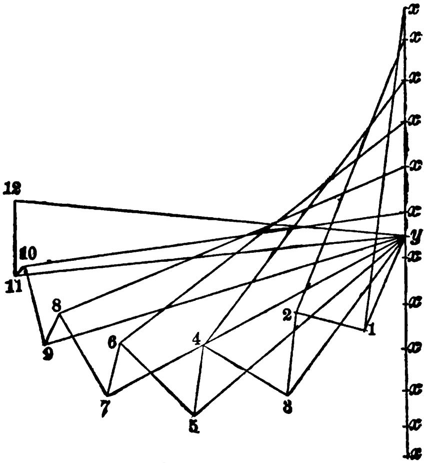

Fig. 49.
P. and S. Diagram.

Or, the quadrant being in this case divided into 6 equal parts by the radials, the inclinations of the roof at the different joints, and the corresponding wind pressures, will be as follows:

Point	A	B	D	F	H	K	M	
Inclination	90.	75.	60.	45.	30.	15.	0	deg.
Wind Pressure	50.0	50.0	50.0	45.0	33.1	17.7	0.0	lbs.

(138.) *Computation of Loads.*

1. *Permanent Loads.*

The weight of the purlines per square foot of roof or per section area varies with their spacing.

Total weight of the entire truss $= 80 \times 4.25 = 5440$ lbs. Hence, average weight for one panel or loaded point, as the entire weight of the truss is assumed to be concentrated at the joints of the upper chord $= 5440 \div 12 = 453$ lbs.

At B or D the tin, sheathing and purlines average 10 lbs. per square foot; P. load per section area $= 167.5 \times 10 + 453 = 2128$ lbs. $= 1.064$ tons.

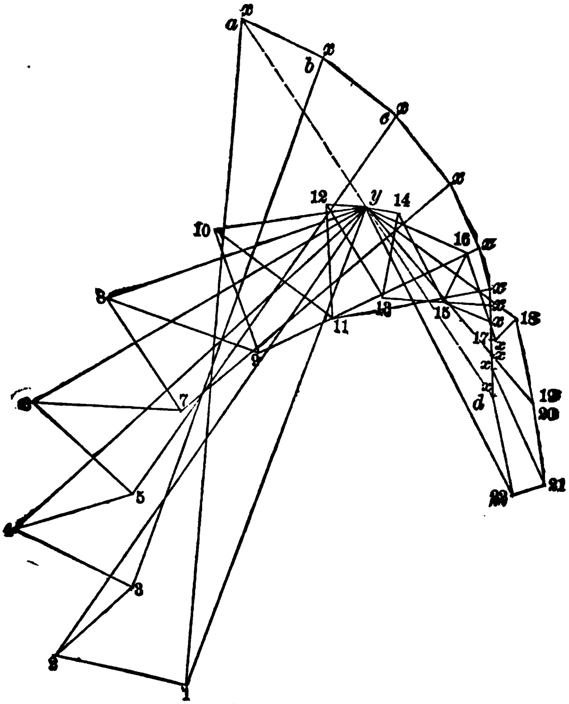

Fig. 50.
P. and W. Diagram.

At F, tin, sheathing and purlines weight 9.5 lbs. per square foot. P. load $= 167.5 \times 9.5 + 453 = 2044$ lbs. $= 1.022$ ton.

At H, tin, sheathing and purlines, weigh 8 lbs. per square foot. P. load $= 167.5 \times 8 + 453 = 1793$ lbs. $= 0.897$ ton.

At K or M, tin, sheathing and purlines weigh $6\frac{2}{3}$ lbs. per square foot. P. load $= 167.5 \times 6\frac{2}{3} + 453 = 1570$ lbs. $= 0.785$ tons.

2. *Snow Loads.*

Bisect each panel length of the upper chord, and drop a vertical to the horizontal line AP; the horizontal distance between any two adjacent verticals $\times 16 =$ horizontal projection of the section area supported at that joint of the upper chord, located between the two verticals; the snow load for that point is found by multiplying the corresponding horizontal projection just found, by 20 lbs., thus proceeding for all the loaded points of the upper chord, obtaining the following snow loads:

At B, $2.73 \times 16 \times 20 = 874$ lbs. $= 0.437$ ton
At D, $5.16 \times 16 \times 20 = 1648$ lbs. $= 0.824$.
At F, $7.44 \times 16 \times 20 = 2380$ lbs. $= 1.190$.
At H, $9.05 \times 16 \times 60 = 2896$ lbs. $= 1.448$.
At K, $10.12 \times 16 \times 20 = 3240$ lbs. $= 1.620$.
At M, $10.47 \times 16 \times 20 = 3350$ lbs. $= 1.675$.

3. *Wind Loads.*

These are computed by multiplying a section area by the normal wind pressure at each joint of the upper chord.

At B, $167.5 \times 50.0 = 8375$ lbs. $= 4.188$ tons.
At D, $167.5 \times 50.0 = 8375$ lbs. $= 4.188$.
At F, $167.5 \times 45.0 = 7538$ lbs. $= 3.769$.
At H, $167.5 \times 33.1 = 5544$ lbs. $= 2.772$.
At K, $167.5 \times 17.7 = 2964$ lbs. $= 1.482$.
At M, no wind pressure and no wind load.

4. *Permanent and Snow Loads.*

At B, $1.064 + 0.437 = 1.501$ tons.
At D, $1.064 + 0.824 = 1.888$.
At F, $1.022 + 1.190 = 2.212$.
At H, $0.894 + 1.448 = 2.342$.
At K, $0.785 + 1.620 = 2.405$.
At M, $0.785 + 1.675 = 2.460$..

To avoid errors, it is well to collect these results in a table of the following form before drawing strain diagrams:

Point	B	D	F	H	K	M
Inclination	75.	60.	45.	30.	15.	0 deg.
W. Pressure	50.0	50.0	45.0	33.1	17.7	0 lbs.
P. Load	1.064	1.064	1.022	0.897	0.785	0.785 tons.
S. Load	0.437	0.824	1.190	1.448	1.620	1.675 tons.
P. and S. Load	1.501	1.888	2.212	2.342	2.405	2.460 tons.
W. Load	4.188	4.188	3.769	2.772	1.482	0. tons.

As the truss is mostly constructed of wood, no expansion rollers are necessary, and it being fixed to each wall, it is not necessary to compute the loads on the half section area supported at A.

(139.) *Strain Diagram for Permanent and Snow Loads.*

In Fig. 49, we lay off downwards on a vertical, the P. and S. loads just found, taken in order from A to M; bisect the load for M at *y*; produce the load line downwards below *y*, and with dividers, take the distance from *y* to each *x*-point above *y*, laying it off below *y*, so as to make the two portions of the load line symmetrical about the point *y*.

Then xy = half the load line = the reaction at either A or P.

For the curved members of the chords, draw the corresponding strain lines parallel to the *chord* of the arc representing the given member; this is most correctly done by drawing a radius to the middle point of the arc, then making the required strain line perpendicular to this radius. (Remember that the centres and radii of the two chords are different.)

If any diagonal member be found to be subject to tension, it should be omitted, the other diagonal of the same panel being used instead, which will then be in compression, as required. Only one diagonal of a panel can be considered at the same time, as a single diagonal divides it in two triangles, making it then perfectly stable, and a second one is not necessary. Besides, two would make the problem indeterminate.

Complete the strain diagram in the manner already described for problems 1 and 3.

(140.) *Strain Diagram for Permanent and Wind Loads.*

At each point B, D, etc., on the windward side of the truss, find the resultant of the permanent and wind loads there acting, as indicated in Fig. 48. (121.)

Commencing at any point *a* (Fig. 50), represent the resultant at

B (Fig. 48), by *ab*, that at D by *bc*, etc., completing the load line *abcd* by taking the resultants in order from A towards P; join *ad*, which represents the resultant of all the forces acting on the truss, as well as the sum of the reactions at A and P; by the method of the equilibrium polygon, divide *ad* into $ay =$ the reaction at A, and $yd =$ the reaction at P.

The diagram is then completed like that for permanent and snow loads, excepting that the dotted diagonals will be required for the windward side of the truss, as they are required to be in compression. Both sets of diagonals would therefore be employed in the actual construction of the truss.

1. Note that all *x*-lines falling on the right of the load line *axd* denote that the corresponding members are subject to tension on the leeward side of the truss, so that the four lower panels of the upper chord are evidently in tension on the leeward side of the roof.

2. Also, that all *y*-lines lying on the right-hand side of a vertical drawn through the *y*-point denote compression in the corresponding members; therefore, the five lower panels of the lower chord are subject to compression on the leeward side of the roof.

(141.) *The Strain Sheet.*

Measure the strain lines of each diagram, and collect the results on a strain sheet, as in the following table:

The names of dotted or counter diagonals are denoted by underlining in column 1.

The greatest regular strain on any member is written in column 5 as a maximum, while the greatest strain of *opposite* nature is written as a minimum in column 6. The corresponding member must be so designed as to safely resist each of these strains acting upon it. (See table on opposite page.)

(142.) *Programme of Conditions for Problem* 5.

Truss to be of type shown in Fig. 51; divided in 7 equal panels, the roof of the middle panel being raised 4 feet to permit the insertion of windows between it and the main roof, for better lighting the centre of the building; truss and roof to be entirely constructed of iron; therefore, the compression members should be shorter than those in tension, to secure proper economy; the long diagonals of the panels are here used instead of the short ones, as in problem 1, because required to be in tension instead of compression.

STRAIN SHEET FOR PROBLEM 4.

MEMBER.	P. & S.	P. & W. W.	P. & W. L.	Maximum.	Minimum.
X 1	−16.20	−34.90	+ 5.63	−34.93	+ 5.63
X 2	−15.25	−29.74	+ 6.68	−29.74	+ 6.68
X 4	−17.55	−24.28	+ 3.40	−24.28	+ 3.40
X 6	−18.87	−18.77	+ 0.50	−18.87	+ 0.50
X 8	−19.68	−13.55	− 2.32	−19.68	
X 10	−20.22	− 8 87	− 5.38	−20.22	
Y 1	+ 5.47	+26.70	−17.13	+26.70	−17.13
Y 3	+10.40	+28.55	−13.44	+28.55	−13.44
Y 5	+14.56	+25.15	− 9.76	+25.15	− 9.76
Y 7	+17.70	+20.47	− 6.00	+20.47	− 6.00
Y 9	+19.72	+14.75	− 1.93	+19.72	− 1.93
Y 11	+20.52	+ 8.62	+ 2.76	+20.52	
1 2	+ 3.78	+ 6.94	− 1.88	+ 6.94	− 1.88
3 4	+ 5.00	+ 6.80	+ 0.00	+ 6.80	
5 6	+ 5.34	+ 7.04	+ 1.40	+ 7.04	
7 8	+ 5.00	+ 7.10	+ 2.73	+ 7.10	
9 10	+ 4.25	+ 6.83	+ 4.15	+ 6.83	
11 12	+ 3.85	+ 5.88	+ 5.88	+ 5.88	
2 3	− 4.25		− 4.58	− 4.58	
2 3		− 5.02		− 5.02	
4 5	− 3.58		− 4.50	− 4.50	
4 5		− 6.35		− 6.35	
6 7	− 2.76		− 4.65	− 4.65	
6 7		− 7.56		− 7.56	
8 9	− 1.80		− 5.05	− 5.05	
8 9		− 7.99		− 7.99	
10 11	− 0.66		− 5.95	− 5.95	
10 11		− 7.48		− 7.48	

Span, 80 feet; rise of upper chord, 16 feet, exclusive of the raised central portion; of the lower chord, 3 feet; the lower chord is made a circular arc, for the sake of appearance and to shorten the verticals and diagonals, though this also increases the strains on the members. The weight of the truss is assumed to be equally divided between all the joints of both chords, as in problem 2, but there is no ceiling.

Roof to be covered with corrugated iron, weighing 2 lbs. per square foot laid, supported by three 6 inch 13.5 lb. I beams for each panel of the main roof; five 6 inch 10 lb. channels being used as purlines for the raised central portion. No expansion rollers are used in this case.

(143.) *Computation of Loads.*

1. *Permanent Loads.*

Length of inclined principal $= \sqrt{16^2 + \left(\frac{3\times80}{7}\right)^2} = 37.83$ feet.

Panel length $= 37.83 \div 3 = 12.61$ feet. Section area $= 12.61\times16 = 201\frac{3}{4}$ square feet. Area supported by one purline $= 201\frac{3}{4} \div 3 = 67\frac{1}{4}$ square feet.

Weight of truss $= 80\times16\times7.38 = 9440$ lbs. It has 14 joints, exclusive of the raised portion; hence, $9440 \div 14 = 674$ lbs. = weight at each joint. (97.)

Corrugated iron $= 201\frac{3}{4}\times2 =$ . . . 403.5 lbs.
Purlines $= 3\times13.5\times16 =$ . . . 648. "
Truss = ... 674. "

Total = 1725.5 lbs. = .868 ton = permanent load at the joints B, D, N and P.

Corrugated iron $= 101\times2 =$ . . . 202. lbs.
Purlines $= 2\times13.5\times16 =$ . . . 432. "
Truss = ... 674. "

Total = 1308 lbs. = .654 ton = permanent load at joints F and L, exclusive of the weight of the raised portion, half of which is supported at each joint F and L.

For the raised central portion:

At F, sash and glass = ... 64 lbs.
Vertical post =. . . . . . 38 "
Total = 102 lbs. = .051 ton.

At H, sash and glass = ... 64 "
Third of small truss = ... 38 "
Purlines $= 1\frac{1}{2}\times16\times10 =$ . . 240 "
Corrugated iron $= 50\frac{1}{2}\times2 =$ . . . 101 "
Total =443 lbs. = .222 ton.

At I, Third of small truss =. . . . . 38 lbs.
Purlines $= 2\times16\times10 =$ . . 320 "
Corrugated iron $= 101\times2 =$ . . . 202 "
Total =560 lbs. = .280 ton.

This makes the total permanent weight of the raised central portion $= .051 + .222 + .280 + .222 + .051 = .826$ ton, which is equally divided between the joints F and L.

Hence, $.654 + .413 = 1.067$ = tons = total permanent load at F and L.

The permanent load at each joint of the lower chord = 674 lbs. = .337 ton.

2. *Snow Loads.*

At joints B, D, F, L, N and P, $= 80 \times 16 \times 20 \div 7 = 3657$ lbs. = 1.829 tons.

3. *Wind Loads.*

Inclination of the roof surface is about 25 deg., so that the normal wind pressure is 28.3 lbs. per square foot.

At B and D, the wind load $= 201\frac{3}{4} \times 28.3 = 5710$ lbs. = 2.855 tons.

At F, the normal wind load $= 101 \times 28.3 = 2855$ lbs. = 1.428 tons.

The horizontal wind load $= 2 \times 16 \times 50 = 1600$ lbs. = .800 ton.

At H, the horizontal wind load $= 2 \times 16 \times 50 = 1600$ lbs. = .800 ton.

At H and I, the normal wind load $= 50.5 \times 28.3 = 1428$ lbs. = .714 ton.

4. *Table of Loads.*

These results may be collected in the following table:

Point.	B	D	F	H	I
P. Load..............	0.868	0.868	1.067	(0.222)	(0.280)
S. Load..............	1.829	1.829	1.829	(0.457)	(0.913)
P. and S..............	2.697	2.697	2.896	(0.679)	(1.193)
W. Normal..........	2.855	2.855	1.428	0.800	0.800
W. Horizontal.......	.....	.....	0.800	0.800	.....

The loads for H and I included in brackets are also included in those given for F.

(144.) *Strain Diagrams for Permanent and Snow Loads.*

1. *Main Truss, exclusive of raised portion.*

On the vertical line *ab*, Fig. 52, lay off downwards the loads at the joints of the upper chord, taken in order from left to right. The middle point *c* of this line will be both an *x*-point and a *y*-point, since the number of panels is odd. From *c* lay off in both directions 3 loads of .337 tons each, representing the weight of the truss at each joint of the lower chord.

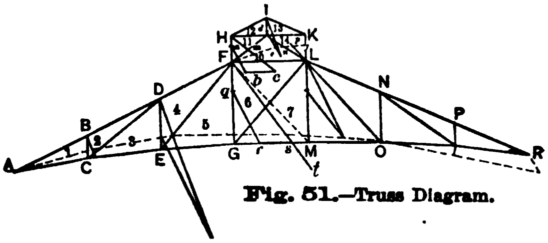

Fig. 51.—Truss Diagram.

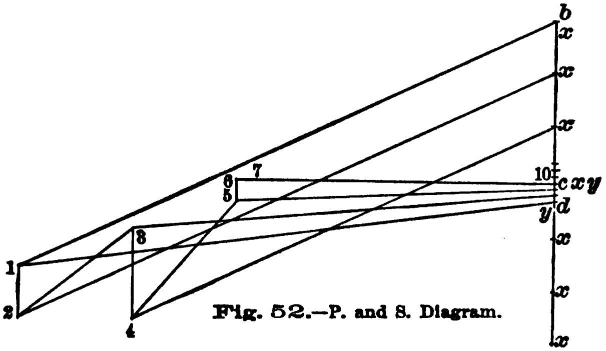

Fig. 52.—P. and S. Diagram.

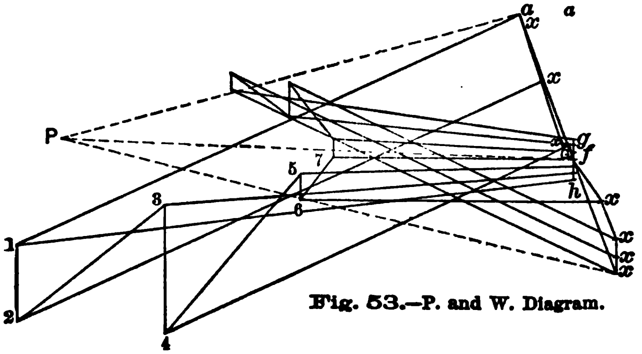

Fig. 53.—P. and W. Diagram.

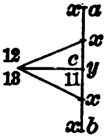

Fig. 54

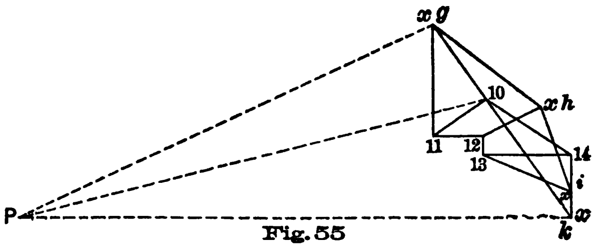

Fig. 55

Diagrams for Raised Portion.

Then for joint A, through *b* and *d* of Fig. 52, draw $x1$ and $y1$, intersecting at 1; for B, draw 1 2 and $x2$, intersecting at 2; for C, draw 2 3 and $y3$, intersecting at 3, etc., completing the strain diagram, of which one-half is here given. Evidently, the strains in 6 10 and $y7$ are equal, as they should be, these members being parallel.

2. *Truss of raised central portion.*

In Fig. 54, lay off the loads acting at H, I and K, downwards from *a* to *b*; bisect *ab* at *c*. Draw $x11$ parallel to $x11$ of Fig. 51, and 11 coincides with *c*, the member being vertical, so that no strains exist on the members 11 10 and 10 14 under permanent and snow loads.

For joint H, draw $x12$ and $y12$, intersecting at 12. The diagram is then drawn for the other side, forming the complete strain diagram as in Fig. 54.

(145.) *Strain Diagram for Permanent and Wind Loads.*

1. *For raised central portion.*

In Fig. 51, make Ha = .222 tons = P. load at H; ab = .714 tons = normal W load; bc = .800 ton = horizontal W load; Hc = their resultant = 1.400 tons.

At I, make Id = .280 ton; de = .714 ton; then Ie = the resultant at I = .970 ton.

In Fig. 55, make gh = 1.400 tons; hi = .970 ton; ik = .222 ton = permanent load at K; then gk = the resultant of these forces. By the method of the equilibrium polygon, divide gk at 10 into $g10$ and $10k$, the respective reactions at F and L.

For joint F, draw $x11$ and 10 11, intersecting at 11; for H, draw 11 12 and $x12$, intersecting at 12, etc. The complete strain diagram is given in the figure.

The strain diagrams of Figs. 54 and 55 are here drawn at a scale five times as large as that of Figs. 52 and 53, for the sake of greater clearness.

2. *Main truss, exclusive of raised portion.*

The resultants at B and D are parallel, and are obtained as in the previous problems.

At F, make Fq = .654+.051 = .705 ton = total permanent load at F, exclusive of the permanent loads at the joints H, I and K; make qr = 1.428 tons = normal W. load; rs = .800 ton =

horizontal W load, and $st = .940$ ton = load at F due to the raised central portion, $= g10$, Fig. 55. The resultant F$t = 3.39$ tons.

At L, make L$u = .705$ ton = P. load at K, exclusive of that of raised portion; $uv = 1.535$ tons = load at L due to raised portion $= k10$, Fig. 55. Resultant L$v = 2.155$ tons.

In Fig. 53, draw the load line *abcde* as before, taking the loads in regular order from left to right, and neglecting the raised central portion, as the loads due to this are included in those at F and L. Then ae = resultant of all the forces acting on the truss. Divide this at f by the method of the equilibrium polygon, into af = reaction at A, and fe = reaction at R.

Through f draw the vertical gh, on which lay off 3 loads upwards and downwards, each $= .337$ ton = load at each joint of lower chord.

For the joint A, draw through a and h, $x1$ and $y1$, intersecting at 1; for B, 1 2 and $x2$, intersecting at 2, etc. The complete strain diagram is here given.

(146.) *The Strain Sheet.*

The Strain Sheet for this Problem is as follows:

STRAIN SHEET.

MEMBER.	P. & S.	P. & W. W.	P. & W. L.	Maximum.	Minimum.
X 1...........	-30.66	-28.75	-22.10	-30.66	
X 2...........	-30.66	-30.09	-22.10	-30.66	
X 4...........	-24.10	-22.90	-18.90	-24.10	
X 11...........	- 1.30	- 1.15	- 0.64	- 1.30	
X 12...........	- 1.40	- 0.63	- 0.98	- 1.40	
6 10............	-16.57	-15.66	-15.66	-16.57	
Y 1............	+28.02	+29.17	+17.86	+29.17	
Y 3............	+21.94	+21.27	+14.88	+21.94	
Y 5............	+16.58	+14.20	+12.40	+16.58	
Y 7............	+16.57	+12.25	+12.25	+16.57	
1 2..........	- 2.70	- 4.05	- 0.87	- 4.05	
3 4..........	- 4.70	- 6.62	- 1.85	- 6.62	
5 6..........	- 1.00	- 1.36	+ 0.95	- 1.36	+ 0.95
12 13..........	-- 0.00	- 0.22	- 0.22	- 0.22	
11 12..........	+ 1.30	- 0.55	+ 0.88	+ 1.30	- 0.55
2 3..........	+ 7.60	+ 9.94	+ 3.70	+ 9.94	
4 5..........	+ 8.20	+10.83	+ 3.78	+10.83	
6 7..........	+ 0.00	+ 3.00	+ 3.00	+ 3.00	
11 10..........	+ 0.00	+ 0.65	- 1.06	- 1.06	+ 0.65

CHAPTER VI.

LENGTHS OF TRUSS MEMBERS.

(147.) In many cases, especially in the construction of iron roofs, it is necessary to correctly determine the length of each member of a truss to the nearest $\frac{1}{1000}$ of a foot or $\frac{1}{100}$ of an inch, a degree of accuracy only attainable by computation, but not possible from measurement of a drawing. In the best Engineering work, the maximum error allowed in the lengths of truss members is $\frac{1}{64}$ inch.

To avoid errors, it would be preferable to make the computations in feet and decimals of a foot, then using a wooden rod for laying off the feet, and a steel scale graduated to $\frac{1}{1000}$ of a foot, for laying off the fractional part of a foot. Or, the decimal of a foot can be changed into inches and fractions, which are most convenient for the ordinary mechanic.

(148.) *Length of a Member.*

The *joint length* of a member here signifies the length of its axis or centre line, taken between the intersections of this axis with the centre lines of those members, which are connected with the ends of the given member. It therefore equals the length of the corresponding line in the truss diagram, this diagram being composed of the centre lines of the different members of the truss. The joint length is always meant in the following formulæ, unless otherwise mentioned.

The *actual length* of any member is seldom the same as its joint length, being either longer or shorter; but is readily determined if the joint length be known, since their differences at each end of the member can easily be measured from the detail drawings of the end joints, which should be made full size, and may be on separate sheets.

The lengths of adjustable members need not be so accurately determined. Tie-rods are usually adjustable by nuts, or by sleeve-nuts placed between their ends; other members are seldom adjustable.

(149.) *Notation employed in Formulæ.*

Let s = span of truss in feet = horizontal distance between centres of end joints; this also usually = distance between centres of the supporting walls.

Let n = number of panels into which the truss is divided. They are of equal horizontal length, unless otherwise stated.

Let r' = rise of upper chord = height of its middle and highest point above a horizontal line drawn through the centres of the end joints of the truss.

Let r'' = rise of the lower chord above the same horizontal.

Let i' = angle of inclination of upper chord, if composed of two straight lines of equal inclination.

Let i' = inclination of a tangent at the end of a curved upper chord.

Let i'' = angle of inclination of lower chord, if composed of two straight lines of equal inclination.

Let i'' = inclination of tangent at end of a curved lower chord.

Let p = number of panels between the *nearest* end of the truss, and any vertical considered, or the *upper* end of any given diagonal.

Let q = number of panels between the *middle* of the truss and any given vertical, or the *upper* end of any diagonal.

Let R' = radius of curvature of a circular upper chord.

Let R'' = radius of curvature of a circular lower chord.

Let d = depth of truss at centre = vertical distance between centre lines of the chords.

Let f = difference between the heights of the ends of any web-member above a horizontal drawn through the centres of the end joints of the truss.

A. General Formulæ.

(150.) *Chord not curved, but composed of two equal principals.*

$\frac{2r'}{s} = \tan i'$ = tan angle of inclination of upper chord.

$\frac{2r''}{s} = \tan i''$ = tan angle of inclination of lower chord.

$\frac{s}{n}$ = joint length of any panel of a horizontal chord, except Fink.

$\frac{s}{n \cos i'}$ = joint length of any panel of inclined upper chord.

$\frac{s}{n \cos i''}$ = length of panel of inclined lower chord, except for a Fink truss.

$r' - r''$ = length of middle vertical, if any, $= d$.

$\frac{2pr'}{n}$ = height of any joint of inclined upper chord above horizontal span line.

$\frac{2pr''}{n}$ = height of any joint of inclined lower chord above horizontal span line.

(151.) *Chord a circular arc.*

$\frac{2r'}{s} = \tan \frac{1}{2} i'$ = tan $\frac{1}{2}$ angle inclination tangent at end of upper chord.

$\frac{2r''}{s} = \tan \frac{1}{2} i''$ = tan $\frac{1}{2}$ angle of inclination at end of lower chord.

$\frac{s}{2 \sin i'} = R'$ = radius of curvature of upper chord.

$\frac{s}{2 \sin i''} = R''$ = radius of curvature of lower chord.

$\frac{\pi s i'}{180 \sin i'}$ = total developed length of upper chord.

$\frac{\pi s i''}{180 \sin i''}$ = total developed length of lower chord.

$2 i$ = angle at the centre of chord subtended by the span s.

B. Special Formulæ.

(152.) *Truss to be of type shown in Fig.* 56 or 57.

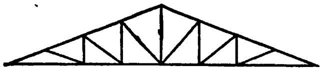

Fig. 56

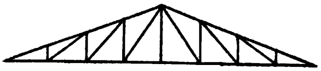

Fig. 57

Lower chord horizontal and straight.

$\frac{2pr'}{n}$ = length of any vertical.

$\frac{1}{n}\sqrt{s^2+4p^2r'^2}$ = length of any diagonal, either truss.

(153.) *Truss of type as in Fig.* 58 *or* 59.

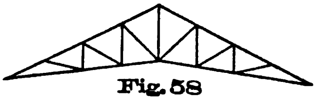

Fig. 58

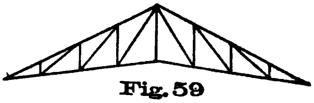

Fig. 59

Lower chord composed of two equal inclined straight lines.

$\frac{2p}{n}(r'-r'')$ = length of any vertical.

$\frac{1}{n}\sqrt{s^2+4\,[p\,(r'-r'')\pm 1]^2}$ = length of any diagonal.

In the last formula use + sign for diagonals inclined inward, as in Fig. 59; use − sign if they incline outward, as in Fig. 58.

(154.) *Truss of type as in Fig.* 60.

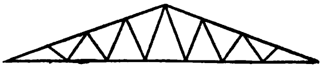

Fig. 60

Upper chord divided in equal parts; joints of lower chord midway between verticals dropped through those of upper and lower chord horizontal.

$\frac{1.5\,s}{n}$ = joint length of end panels of lower chord.

$\frac{1}{n}\sqrt{\frac{s^2}{4}+4p^2r'^2}$ = length of any web-member.

(155.) *Truss of type shown in Fig.* 61.

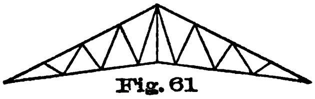

Fig. 61

Similar to the last, except that lower chord is composed of two equal inclined lines.

$\frac{1.5\,s}{n\cos i''}$ = length of end panels of lower chord.

$\frac{1}{n}\sqrt{\frac{s^2}{4}+4\,[p\,(r'-r'')\pm r'']^2}$ = length of any web-member.

Use + sign for members inclined at top towards centre of truss; — sign for those inclined outwards.

(156.) *Fink truss of type as in Fig.* 62.

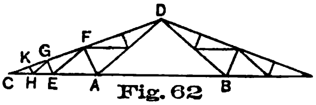

Fig. 62

Lower chord entirely horizontal.

$\frac{2r'}{\tan 2i'}$ = length of middle portion AB of lower chord.

$\frac{r'}{\sin 2i'}$ = length of primary tie-rod AC, AD, etc.

One-half this = length of secondary tie-rod AE, CE, etc.

One-fourth this = length of tertiary tie-rod CH, etc.

$\frac{s \tan i'}{4 \cos i'}$ = length of primary strut AF, etc.

One-half this = length of secondary strut EG, etc.

One-fourth this = length of tertiary strut HK, etc.

(157.) *Fink truss of type as in Fig.* 63.

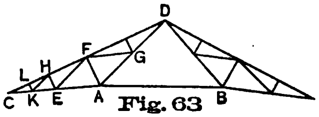

Fig. 63

Central portion AB horizontal, but raised above a horizontal through end joints of truss.

$\frac{r''}{\frac{r'}{\sin 2i'} - r'' \tan i'} = \tan i''$ = tan inclination of the portion AC of lower chord.

$\frac{2(r'-r'')}{\tan (2i'-i'')}$ = length of horizontal portion AB of lower chord.

$\frac{r'-r''}{\sin (2i'-i'')}$ = length of primary tie-rod AC, AD, etc.

One-half this = length of secondary tie-rod CE, EF, etc.

One-fourth this = length of tertiary rod CK, HK, etc.

$\frac{\sin(i'-i'')(r'-r'')}{\sin(2i'-i'')}$ = length of primary strut AF, etc.

One-half this = length of secondary strut EH, etc.

One-fourth this = length of tertiary strut KL, etc.

$\frac{\tan i''(r'-r'')}{\tan 2(i'-i'')}$ = distance from the horizontal line AB, up to the intersection of AC produced, with the middle vertical through D.

(158.) *Fink truss of type as in Fig.* 64.

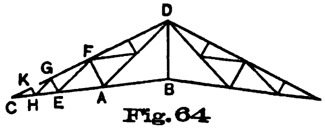

Fig. 64

Lower chord composed of two lines of equal inclination.

$\frac{\cos(2i'-i'')(r'-r'')}{\sin(2i'-2i'')}$ = length of portion AB of lower chord.

$\frac{s}{4\cos i' \cos(i'-i'')}$ = length of primary tie-rod AC, etc.

One-half this = length of secondary rod CE, etc.

One-fourth this = length of tertiary rod CH, etc.

$\frac{s\tan(i'-i'')}{4\cos i'}$ = length of primary strut AF, etc.

One-half this = length of secondary strut EG, etc.

One-fourth this = length of tertiary strut KH, etc.

(159.) *Truss of type as in Fig.* 65 or 66.

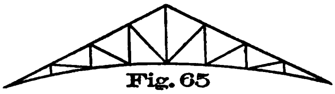

Fig. 65

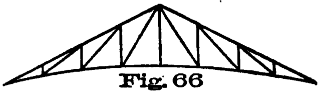

Fig. 66

Lower chord a circular arc; upper chord as before.

$2i''$ = angle subtended at centre of lower chord by span s.

$\frac{2pr'}{n}$ = height of any joint of upper chord above a horizontal drawn through the end joints of the truss.

$r''-R''+\sqrt{R''^2-\left(\frac{qs}{n}\right)^2}$ = height of any joint of lower chord above the same span line.

f = difference of height of its ends $=\underline{2\,pr'}-r''+\mathrm{R}''-\sqrt{\mathrm{R}''^2-\left(\frac{qs}{n}\right)^2}$ = length of any vertical. (149.)

$\sqrt{\frac{s^2}{n^2}+f^2}$ = length of any diagonal, inclined either way, or length of any panel of lower chord, measured on a straight line and not on the arc itself.

(160.) *Truss of type shown in Fig.* 67.

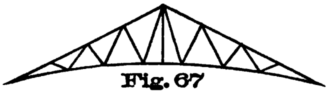

Fig. 67

Same as in the last case, except that the joints of lower chord are midway between verticals dropped through those of upper chord.

$\sqrt{\left(\frac{1.5s}{n}\right)^2+f^2}$ = length of end panels of lower chord, straight.

$\sqrt{\left(\frac{s}{2n}\right)^2+f^2}$ = length of any half panel of lower chord, straight.

Other formulæ are as in the last case (159.)

(161.) *Truss of type as in Fig.* 68 or 69.

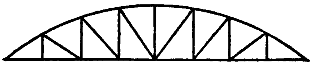

Fig. 68

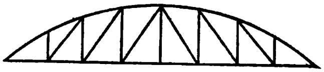

Fig. 69

Diagonals inclined either way; lower chord horizontal; panels of equal horizontal length.

$r'-\mathrm{R}'+\sqrt{\mathrm{R}'^2-\left(\frac{qs}{n}\right)^2}$ = height of any joint of upper chord above the horizontal lower chord.

The same = length of the corresponding vertical.

Let V = length of a vertical through the upper end of any diagonal.

$\sqrt{\frac{s^2}{n^2}+V^2}$ = length of this diagonal.

$\sqrt{\frac{s^2}{n^2}+f^2} \doteq$ straight length of any panel of upper chord.

(162.) *Truss of type as in Fig.* 70 *or* 71.

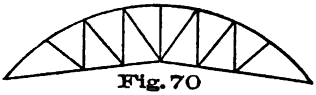

Fig. 70

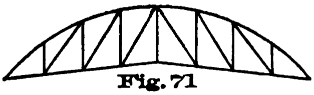

Fig. 71

$r'-R'+\sqrt{R'^2-\left(\frac{qs}{n}\right)^2}$ = height of any joint of upper chord above a horizontal through centres of end joints.

$\frac{2\,pr''}{n}$ = height of any joint of lower chord above same horizontal.

f = length of any vertical.

$\sqrt{\frac{s^2}{n^2}+f^2}$ = length of any diagonal, or straight length of any panel of upper chord.

(163.) *Truss as in Fig.* 72 *or* 73.

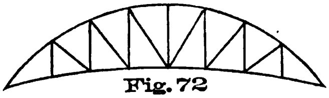

Fig. 72

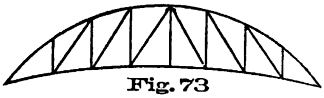

Fig. 73

Both chords are circular arcs; panels of equal horizontal length.

$r'-R'+\sqrt{R'^2-\left(\frac{qs}{n}\right)^2}$ = height of any joint of upper chord above horizontal span line.

$r''-R''+\sqrt{R''^2-\left(\frac{qs}{n}\right)^2}$ = height of any joint of lower chord above same horizontal.

f = length of any vertical.

$\sqrt{\frac{s^2}{n^2}+f^2}$ = length of any diagonal, or straight length of any panel of either chord.

(164.) *Truss as in Fig.* 74 *or* 75.

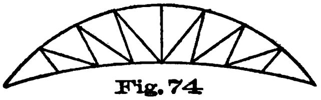

Fig. 74

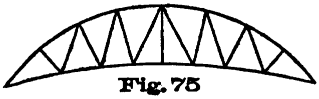

Fig. 75

Both chords are circular arcs; upper chord divided into panels of equal length on the curve by radials drawn to its centre.

$2\,R'\sin\left(\frac{i'}{n}\right)$ = straight joint length of any panel of upper chord.

Let $a=\cot\left(\frac{2\,qi'}{n}\right)$

Let $b=(R''+r'-r'')$.

$r'-R'\left[1-\cos\left(\frac{2\,qi'}{n}\right)\right]$ = height of any joint of upper chord above a horizontal through end joints of truss.

$\frac{r'-a^2b+R'}{1+a^2}\pm\sqrt{\frac{a^2R''^2-a^2b^2-R'^2+}{1+a^2}\left(\frac{a^2b^2+R'^2}{1+a^2}\right)^2}$ = height of any joint of lower chord above the same horizontal.

$R'\left[1-\sin\left(\frac{2\,qi'}{n}\right)\right]$ = horizontal distance from a vertical drawn through centre of truss, to any joint of the upper chord.

$\frac{a\,(b-R')}{1+a^2}\pm\sqrt{\frac{R''^2-R'^2+2\,b\,R'-b^2}{1+a^2}+\left(\frac{ab-aR'}{1+a^2}\right)^2}$ = horizontal distance from the same vertical, to any joint of the lower chord.

Let f = difference between vertical heights of the ends of any member, above a horizontal span line, as before.

Let g = difference between the horizontal distances to the ends of the same member, from a vertical through the centre of the truss.

$\sqrt{f^2+g^2}$ = length of any radial or diagonal, or the straight length of any panel of either chord.

The truss diagram is in this case usually drawn full size, the lengths of the members being then measured on it. But it is preferable to find them by computation, though the process is rather lengthy.

(165.) *Crescent Truss as in Fig.* 76 *or* 77.

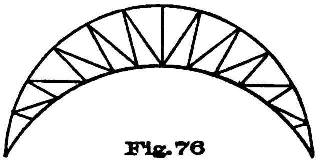

Fig. 76

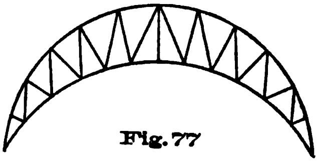

Fig. 77

Upper chord a semicircle, divided in equal panels by radials.

Here $i' = 90°$; $R' = r' = \frac{s}{2}$

$\frac{\pi s}{2}$ = total length of upper chord.

$\frac{\pi s}{2n}$ = curved panel length of upper chord.

$s \sin\left(\frac{90}{n}\right)$ = straight panel length of upper chord.

$\frac{s}{2} \cos\left(\frac{180q}{n}\right)$ = height of any joint of upper chord above a horizontal span line.

$$\frac{s}{2} - \frac{a^2 b + \frac{s}{2}}{1 + a^2} \pm \sqrt{\frac{a^2 R''^2 - a^2 b^2 - \frac{s^2}{4}}{1 + a^2} + \left(\frac{a^2 b^2 + \frac{s}{2}}{1 + a^2}\right)^2} =$$

height of any joint of the lower chord above span line.

$\frac{s}{2}\left[1 - \sin\left(\frac{180\,q}{n}\right)\right]$ = horizontal distance from a vertical through centre of truss, to any joint of upper chord.

$$\frac{a\left(b - \frac{s}{2}\right)}{1 + a} \pm \sqrt{\frac{R''^2 - \frac{s^2}{4} + bs - b^2}{1 + a^2} + \left(\frac{ab - \frac{as}{2}}{1 + a^2}\right)^2} = \text{hori-}$$

zontal distance from same vertical to any joint of lower chord.

$\sqrt{f^2 + g^2}$ = length of any radial, diagonal, or straight length of any panel of either chord.

Here, f = difference in height of ends of the member, and g = difference in their distances from the middle vertical.

(166.) *Truss as in Fig.* 78 *or* 79.

Both chords semi-circular and concentric.

Let d = depth of truss at centre.

$\frac{s}{2} = R'$ = radius of upper chord.

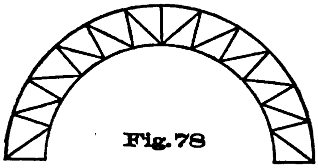
Fig. 78

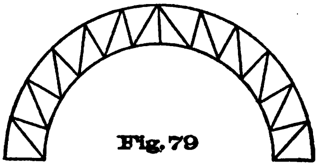
Fig. 79

$\frac{\pi s}{2}$ = total length of upper chord.

$\frac{\pi s}{2n}$ = curved panel length of upper chord.

$s \sin\left(\frac{90}{n}\right)$ = straight panel length of upper chord.

$\frac{s}{2} - d = R''$ = radius of lower chord.

$\pi\left(\frac{s}{2} - d\right)$ = total length of lower chord.

$\frac{\pi}{d}\left(\frac{s}{2} - d\right)$ = curved panel length of lower chord.

$\left(\frac{s}{2} - d\right)\left[2 \sin\left(\frac{90}{n}\right)\right]$ = straight panel length of lower chord.

The panels are all similar, and each may be divided in two triangles by either diagonal.

Taking the outer triangle, its radial side $= d$; the chord side $= s \sin\left(\frac{90}{n}\right)$; the angle included between these sides $= 90° - \frac{90°}{n}$

The length of the diagonal may then be computed by means of the ordinary trigonometrical formulæ for an oblique-angled triangle, having two sides and the included angle given, to find the other side.

(167.) *Truss as in Fig.* 80 *or* 81.

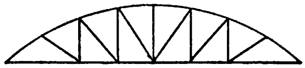
Fig. 80

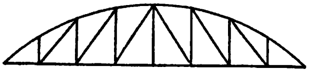
Fig. 81

Lower chord horizontal, upper chord a parabola, whose vertex is at its centre; panels of equal horizontal length.

r' = length of middle vertical.

$r'\left(1 - \frac{4q^2}{n^2}\right)$ = length of any other vertical.

$r' - \frac{4q^2r'}{n^2}$ = height of any joint of upper chord above horizontal span line.

$\sqrt{\frac{s^2}{n^2} + f^2}$ = length of any diagonal, or straight length of any panel of upper chord.

(168.) *Truss as in Fig.* 82 *or* 83.

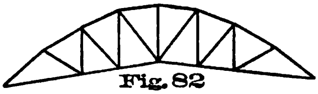

Fig. 82

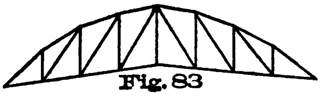

Fig. 83

Upper chord parabolic, vertex at centre; lower chord composed of two lines of equal inclination.

$r' - \frac{4q^2r'}{n^2}$ = height of any joint of upper chord above horizontal span line.

$r'' - \frac{qr''}{n}$ = height of any joint of lower chord.

f = length of any vertical.

$\sqrt{\frac{s^2}{n^2} + f^2}$ = length of any diagonal, or panel of upper chord.

(169). *Truss as in Fig.* 84 *or* 85.

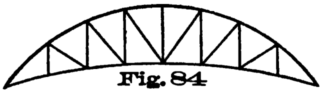

Fig. 84

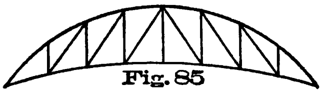

Fig. 85

Both chords parabolic, vertices at their centres.

$r' - \frac{4q^2r'}{n^2}$ = height of any joint of upper chord above horizontal span line.

$r'' - \frac{4q^2r'}{n^2}$ = height of any joint of lower chord.

f = length of any vertical.

$\sqrt{\frac{s^2}{n^2} + f^2}$ = length of any diagonal, or panel of either chord.

CHAPTER VII.

FORMULAE AND TABLES.

GENERAL EXPLANATIONS.

(170.) The following formulæ and tables are required for determining the sectional dimensions of the elementary parts of roofs, such as rafters, ceiling joists, purlines, members of trusses, details of joints, etc.

The formulæ are really derived from those formulæ usually given in works on the resistance of materials, their different appearance resulting from the fact that lengths are given in feet instead of inches, and loads or strains are given in tons of 2,000 pounds, the numerical co-efficients being changed accordingly. This makes the use of the formulæ much more simple, especially in calculations made without the aid of logarithms.

Examples of the use of the formulæ will be given in Chapter VIII.

(171.) *Kinds of Strain.*

There are five principal kinds of strain which may act on the different members of architectural and engineering structures:

1. *Tension*, which acts lengthwise, tending to stretch the member affected.

2. *Shearing*, which tends to slide one part on the other, along a plane of shear or separation.

a. Transverse shear acts across the fibres of wood, or in any direction in case of other materials.

b. Longitudinal shear acts parallel with the fibres of wood only.

3. *Compression* may act in any direction, but always tends to compress or shorten the member in that direction. If the piece be short, it is often termed *Crushing*.

a. Transverse crushing acts perpendicular to the fibres of wood only.

b. Simple crushing acts lengthwise the fibres of wood; in any direction in other materials; but the member must be so short as to give way by crushing alone.

c. Mixed crushing and bending, when the piece is of medium length, giving way partly by crushing, partly by bending.

d. Simple bending, if the piece be sufficiently long to fail entirely by bending, not by crushing.

Most columns, posts and struts belong to the two last.

4. *Transverse Strain* usually acts across the member, being caused by a load supported by the member; the member is usually horizontal, though it is often inclined, like a rafter. The beam generally fails by the rupture of the lower or the crushing of the upper fibres, at its centre.

This failure may occur in either of two ways, both of which must be considered in any given case:

a. By *Breaking*, which is avoided by the use of a sufficient factor of safety.

b. By *Bending* so much as to become unsightly, or to crack plastering; avoided by limiting the amount of deflection.

5. *Torsion* tends to twist off a member, as in turning a nut with a wrench. It seldom occurs in architectural or engineering structures, after they are once completed, and does not therefore require further consideration here; but it is of great importance in mechanical engineering, being one of the strains most commonly found in machines.

(172.) These strains may be further classified as follows:

a. *Direct*, whose intensity is assumed to be equal over the entire resisting area of the member; the corresponding formulæ are all alike and very simple.

Tension, Shearing, Compression (short pieces).

b. *Indirect*, which produce other forms of strain, or are unequally distributed over the resisting area.

Compression (pieces of medium or great length), Transverse Strain, Torsion.

(173.) *Notation employed.*

Let A = sectional area of piece in square inches.
b = greater side of rectangular section in inches.
a = lesser side of rectangular section in inches.

Let s = side of square section in inches.
d = diameter of circular section in inches.
Z = maximum safe resistance of piece in net tons.
f = factor of safety, usually = 5.
f = 10 for spliced wooden tie-beams.
f = 2.5 for resistance of woods to transverse crushing.

Formulæ for Tension.

(174.) *Any Form of Cross Section.*

Let T = co-efficient for tension, for the given material, = ultimate tensile resistance of a bar 1 inch square, in net tons. For values, see (233).

$Z = \frac{A\,T}{f}$ = maximum safe tensile strength of piece in tons.

$A = \frac{Zf}{T}$ = minimum safe sectional area of piece, square inches.

(175.) *Rectangular Cross Section.*

$Z = \frac{ab\,T}{f}$ = maximum safe tensile strength, tons.

$a = \frac{Zf}{Tb}$ = minimum safe thickness, inches.

$b = \frac{Zf}{Ta}$ = minimum safe breadth, inches.

(176.) *Square Cross Section.*

$Z = \frac{s^2 T}{f}$ = maximum safe strength, tons.

$s = \sqrt{\frac{Zf}{T}}$ = minimum safe side, inches.

(177.) *Circular Cross Section.*

$Z = \frac{\pi d^2 T}{4f}$ = maximum safe strength, tons.

d = nearly $1\frac{1}{8}\sqrt{\frac{Zf}{T}}$ = minimum safe diameter, inches.

Bolts or Rods of Round Wrought Iron.

(178.) A. *Screw Ends not enlarged.* See "American Architect," No. 401. With heads, nuts and washers. Table 1.

The maximum safe strength of the finished rod will be less than that of the original rod, on account of being weakened by cutting the screw threads, and this strength is therefore to be determined by Table 1, and not by the formulæ of (177).

1. Maximum safe strength of rod given; required, least safe diameter of rod, dimensions of head, nut, washers, etc.

Look for the given safe strength, or the next larger value, in column 2 of Table 1; the required diameter of rod and the other dimensions will be found on the same horizontal line, and in the proper columns of the Table.

2. Diameter of rod given; required, its maximum safe strength.

Look for given diameter in column "Diam.;" on the same horizontal line its required maximum strength will be found in column 2.

(179.) B. *Screw Ends enlarged.* Table 2.

The maximum safe strength equals that of the original rod, if the work be properly done, and may be found by formula (177), or more conveniently by means of Table 2, which is to be used in the manner already explained for Table 1 (178).

The rod with enlarged ends has a nut on each end; it should be employed whenever possible, because more economical than that with ends not enlarged.

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Formulæ for Shearing.

Notation.

Let S = co-efficient for shearing for the given material, across the fibres of wood, or in any direction in other materials, = resistance to shearing of an area of one square inch in net tons.

Let S′ = co-efficient for shearing parallel with the fibres of wood only.

A. *Shearing in all cases, except parallel with fibres of wood.*

(180.) *Any Form of Cross Section.*

$$Z = \frac{A\,S}{f} = \text{maximum safe resistance in tons.}$$

$$A = \frac{Z\,f}{S} = \text{minimum safe area of piece, square inches.}$$

(181.) *Rectangular Cross Section.*

$Z = \frac{abS}{f}$ = maximum safe resistance, tons.

$a = \frac{Zf}{Sb}$ = minimum safe thickness, inches.

$b = \frac{Zf}{Sa}$ = minimum safe breadth, inches.

(182.) *Square Cross Section.*

$Z = \frac{s^2S}{f}$ = maximum safe resistance, tons.

$s = \sqrt{\frac{Zf}{s}}$ = minimum safe side, inches.

(183.) *Circular Cross Section.*

$Z = \frac{\pi d^2 S}{4f}$ = maximum safe resistance, tons.

$d = 1\frac{1}{8}\sqrt{\frac{Wf}{S}}$ nearly, = minimum safe resistance, tons.

B. *Shearing parallel with fibres of wood.*

(184.) *Any form of Cross Section.*

$Z = \frac{AS'}{f}$ maximum safe resistance, tons.

$A = \frac{Zf}{S'}$ = minimum safe area, square inches.

(185.) *Rectangular Cross Section.*

$Z = \frac{abS'}{f}$ = maximum safe resistance, tons.

$a = \frac{Zf}{S'b}$ = minimum safe length of piece, inches.

$b = \frac{Zf}{S'a}$ = minimum safe breadth, inches.

(186.) *Circular Cross Section.*

$Z = \frac{\pi d^2 S'}{4f}$ = maximum safe resistance, tons.

$d =$ nearly $1\frac{1}{8}\sqrt{\frac{Zf}{S'}}$ = minimum safe diameter, inches.

Formulæ for Compression.

(187.) *Notation employed.*

Let $C =$ co-efficient for crushing in all cases, except across fibres of wood, $=$ ultimate resistance to crushing per square inch in tons.

Let $C' =$ co-efficient for crushing across fibres of wood only.

Let $k =$ factor of safety in this case only, $= 2.5$, except for washers in roof trusses, then $= 1.25$.

Let $m =$ length of end bearing of timbers on walls, etc., inches.

Let $b =$ horizontal breadth or thickness of timbers, inches.

Let $W =$ total load on timber, at centre or uniform, tons.

Let $c =$ distance between centres of joists, rafters, etc., inches.

Let $L =$ length of timbers in feet.

Let $w =$ total weight of one square foot of floor, roof, etc., and its maximum load, in tons.

Let $A =$ sectional or solid area of a column, square inches.

Let $f = 6$ for cast-iron columns, $= 4$ for those of wrought iron; $= 5$ for wooden struts, etc.; $= 8$ to 10 for masonry piers.

Let $d =$ external diameter of a hollow cylindrical column, inches.

Let $Rg =$ radius of gyration of cross section of an iron column or strut, inches. (68.)

Let $P =$ maximum safe compression or load on column or strut, per square inch of section, in tons.

A. *Simple Crushing.*

Length of piece limited to a few times the least dimension of its cross section.

a. Crushing in all cases, except across fibres of wood.

(188.) *Any Form of Cross Section.*

$Z = \dfrac{AC}{f} =$ maximum safe resistance, tons.

$A = \dfrac{Zf}{C} =$ minimum safe sectional area, square inches.

(189.) *Rectangular Cross Section.*

$Z = \dfrac{abC}{f} =$ maximum safe resistance, tons.

$a = \dfrac{Zf}{Cb} =$ minimum safe thickness, inches.

$b = \frac{Zf}{Ca}$ = minimum safe breadth, inches.

(190.) *Square Cross Section.*

$Z = \frac{s^2C}{f}$ = maximum safe resistance, tons.

$s = \sqrt{\frac{Zf}{C}}$ = minimum safe side, inches.

(191.) *Circular Cross Section.*

$Z = \frac{\pi d^2 C}{4f}$ = maximum safe resistance, tons.

d = nearly $1\frac{1}{8}\sqrt{\frac{Zf}{C}}$ = minimum safe diameter, inches.

b. Crushing across fibres of wood only.

Let k = factor of safety for this case.

(192.) *Any Form of Cross Section.*

$Z = \frac{AC'}{k}$ = maximum safe resistance, tons.

$A = \frac{Zk}{C'}$ = minimum safe sectional area, square inches.

(193.) *Rectangular Section or Area.*

$Z = \frac{abC'}{k}$ = maximum safe resistance, tons.

$a = \frac{Zk}{C'b}$ = minimum safe thickness, or length of area, inches.

$b = \frac{Zk}{C'a}$ = minimum safe breadth, inches.

(194.) *Square Cross Section, or Area.*

$Z = \frac{s^2C'}{k}$ = maximum safe resistance, tons.

$s = \sqrt{\frac{Zk}{C'}}$ = minimum safe side, inches.

(195.) *Circular Section or Area.*

$Z = \frac{\pi d^2 C'}{4k}$ = maximum safe resistance, tons.

d = nearly $1\frac{1}{8}\sqrt{\frac{Zk}{C'}}$ = minimum safe diameter, inches.

(196.) *End-Bearing of Beam, Girder, etc.*

$$m = \frac{Wk}{2C'b} = \text{minimum safe length of bearing, inches.}$$

Also apply formula (189) to determine safe resistance of wall to crushing, taking $a = m$ just found by (196). Z in formula (189) must be the actual pressure of the end of the timber on the wall, not more than one-half W in (196).

(197.) *End-Bearing of Joists, Rafters, etc., 2 inches thick.*

$$m = \frac{wcLk}{48C'} = \text{minimum safe end-bearing, inches.}$$

B. Mixed Crushing and Bending.

Pieces of medium length. The same formulæ are now generally employed as in case of simple bending. (C.)

C. *Simple Bending. Long Columns, Struts, etc.*

The formulæ are much more complex than those for A and B, being partly theoretical, but principally based on the results of experiments on columns.

(198.) *Solid Columns, Struts, etc.*

These are usually of wood, those of metal being either hollow, or built up of bars and plates, riveted together to secure strength and economy of material.

The maximum safe or working load Z is usually given to obtain the least safe dimensions of the cross section of the column.

There are two general modes of procedure:

a. By Computation.

(199.) Assume dimensions of cross section, compute by the proper formula the corresponding value of P = maximum safe compression per square inch of sectional area; then Z = PA = total maximum safe load or compression on the column.

Should this differ materially from the given value of Z, repeat the process until a section is found which gives practically the same value.

b. By the use of Graphical Tables, as explained hereafter.

(200.) *Square Posts of White Oak.*

$$P = \frac{3.00}{f\left(1 + .576\,\frac{L^2}{s^2}\right)} = \text{maximum safe load per square inch.}$$

Square Posts of White Pine.

$$P = \frac{2.5}{f\left(1 + .576\frac{L^2}{s^2}\right)} = \text{maximum safe load per sq. inch.} \quad (203.)$$

(201.) *Rectangular Posts of White Pine.*

Let a = thickness or least side of post, inches.

Let b = breadth or greater side, inches.

1. Given, a, L, f and Z; required, b.

By (200), or more conveniently by (203), determine maximum safe strength of a square column, whose side $s = a$; call this W′.

Then $b = \frac{aZ}{W'}$ = required breadth of column.

2. Given, b, L, f and Z; required, a.

Assume a value of a, and proceed, as in the last case, to determine the greater side b; should this differ materially from the given value of b, repeat the process.

(202.) *Rectangular Posts of White Oak.*

Proceed as in (201), but taking $f = 4\frac{1}{8}$ instead of 5.

(203.) *Graphical Table* 3, *for Square Posts of White Pine.*

This table is computed by means of the formula for square pine posts (200), using a factor of safety = 5.

1. Given, L, maximum safe load Z; required, safe side s.

Look for intersection of a vertical through Z, with a horizontal through L. Should this fall on a curve, the corresponding number at the end of the curve will be the required side s; if it falls between two curves, estimate value of the side to nearest $\frac{1}{8}$ inch, according to relative distance of the point from the two nearest curves between which it lies. It will usually be necessary to take the next larger side in even inches, since dimensions of timber are usually in multiples of 2 inches.

(204.) 2. Given, L, and side s; required, maximum safe load Z.

Look for intersection of a horizontal through L, with the curve representing s; a vertical through this intersection gives safe load Z at top of the table.

(205.) *Columns of Cast Iron.*

Section a solid cross or ring of metal, not built up.

a. Hollow Cylindrical.

$$P = \frac{40.}{f\left(1 + .36\,\frac{L^2}{d^2}\right)} = \text{maximum safe load per square inch.}$$

b. Hollow Square.

$$P = \frac{40}{f\left(1 + .27\,\frac{L^2}{s^2}\right)} = \text{maximum safe load per square inch.}$$

c. Star or Cross Columns.

$$P = \frac{40}{f\left(1 + 1.08\,\frac{L^2}{b^2}\right)} = \text{maximum safe load per sq. in., tons.}$$

(206.) *Mode of using preceding Formulæ.*

Given Z, f, L and d, s or b; required, the thickness of metal.

By proper formula, determine P; then $Z \div P =$ minimum safe sectional area in square inches $= A$. Compute total area of the section of the column $= A'$; then $A' - A =$ area of hollow. Compute diameter or side of hollow, d' or s. Then $\frac{1}{2}(d - d')$ or $\frac{1}{2}(s - s') =$ required thickness of metal.

For star columns, $Z \div P = A =$ area of section.

Then $t = b - \sqrt{b^2 - A} =$ thickness of the metal.

Cast iron is now seldom used in the construction of roofs.

(207.) *Columns or Struts of Wrought Iron.*

Columns or struts composed of bars or beams riveted together.

The following formulæ have been much used in practice, though the tables of the Pencoyd Iron Co. are preferable.

Let $\mathrm{R}g =$ radius of gyration of cross section in inches.

a. Both ends of columns or strut flat.

$$P = \frac{20}{f\left(1 + .004\,\frac{L^2}{\mathrm{R}g^2}\right)} = \text{maximum safe load per sq. inch, tons.}$$

b. One end flat, the other pin-jointed.

$$P = \frac{20}{f\left(1 + .006\,\frac{L^2}{\mathrm{R}g^2}\right)} = \text{maximum safe load per sq. inch, tons.}$$

c. Both ends pin-jointed.

$$P = \frac{20}{f\left(1 + .008\,\frac{L^2}{\mathrm{R}g^2}\right)} = \text{maximum safe load per sq. inch, tons.}$$

(208.) *Mode of using the preceding Formulæ.*

Given, L, Z and f (usually $= 4$ for wrought-iron columns).

Assume a section for the column and determine the least value of Rg for that section, for an axis passing through the centre of gravity of the column or strut. This may usually be done by the aid of the values of Rg for beam sections, given in Carnegie's Pocket Book, etc., or by the formulæ of (73), and the general formula of reduction (70).

(209.) *Graphical Table No.* 4.

This is empirical, being based on the results of experiments on different forms of section of wrought-iron beams made by the Pencoyd Iron Co. It offers the latest and best mode of determining the strengths of sections of wrought-iron struts, principals, etc.*

The horizontal scale at top represents the values of L÷Rg, L being in feet, Rg in inches; the vertical scale at the left gives the corresponding safe compression or load per square inch of the metal cross section.

There are four curves, corresponding: 1, to struts with both ends rounded or hemispherical; 2, ends hinged or pin-jointed; 3, ends flat; 4, ends fixed.

Given, L and Z; required, the dimensions of one or more I-beams, channels, T or angle bars, required to safely resist the compression Z.

Assume a beam of the required form of section, and find the corresponding value of Rg and area of cross section A, from any of the Iron Mill Books.

Look for intersection of a vertical through L÷Rg, with the curve corresponding to the manner in which the ends of the member are attached to adjacent members; a horizontal through this intersection gives the maximum safe compression P per square inch of the section, at the left side of the table. Then PA = safe resistance of the assumed beam.

Should PA differ materially from the given value of Z, repeat the process, assuming a larger or smaller beam, as required. It is usually necessary to take the lightest or most economical beam.

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If the beam or strut be free to give way in either direction, the smallest value of the Rg for the section must be taken.

If a member be composed of two I-beams, channels, etc., placed side by side and laced together by diagonal bars, the minimum distance between the centres of gravity of the two beams must not be less than twice the value of Rg for one beam, at right angles to this distance.

The maximum distance between lacing points on either beam must not exceed (length of the member) (least Rg for one beam) ÷ (least Rg for the compound member).

Formulæ for Transverse Strain.

(210.) *Explanations.*

The beam or piece may fall in either of two ways, both of which must be considered. Both kinds of formulæ must be applied, and the *safest* result taken as the true one.

a. By *Breaking*, the beam fails by giving way at the centre.

b. By *Bending*, the beam fails by deflecting too much.

(211.) *Notation.*

Let W = total load supported by the beam or piece, in tons.

Let L = clear length of the piece or beam, in feet.

Let I = moment of inertia of cross section of beam. (66.)

Let r = maximum safe deflection per foot of length of beam, usually = .03 inch.

Let b = horizontal breadth or thickness of rectangular section in inches.

Let d = depth of rectangular section, inches.

Let f = factor of safety, usually = 5.

Let B = co-efficient for breaking for the given material, = centre breaking load of bar 1 inch square, 12 inches long between supports of ends.

Let E = co-efficient for bending for the material = its modulus of elasticity ÷ 2,000.

Let d' = vertical distance from neutral axis of section to most distant fibre of section, in inches.

Let M = maximum bending moment acting on the beam, usually found graphically. (50.)

A. *Beam supported at Ends, Load concentrated at Centre.*

(212.) *Cross Section not rectangular.*

BREAKING.	BENDING.
$W=\frac{6BI}{Lfd'}$	$W=\frac{EIr}{36L^2}$
$I=\frac{WLfd'}{6B}$	$I=\frac{36WL^2}{Er}$
$L=\frac{6BI}{Wfd'}$	$L=\sqrt{\frac{EIr}{36W}}$

(213.) *Cross Section rectangular, solid.*

BREAKING.	BENDING.
$W=\frac{Bbd^2}{fL}$	$W=\frac{Ebrd^3}{432L^2}$
$b=\frac{WfL}{Bd^2}$	$b=\frac{432WL^2}{Erd^3}$
$d=\sqrt{\frac{WfL}{Bb}}$	$d=\sqrt[3]{\frac{432WL^2}{Ebr}}$
$L=\frac{Bbd^2}{Wf}$	$L=\sqrt{\frac{Ebrd^3}{432W}}$

B. *Beam supported at ends, load uniform.*

(214.) *Section not rectangular.*

BREAKING.	BENDING.
$W=\frac{12\,BI}{Lfd'}$	$W=\frac{EIr}{22.5L^2}$
$I=\frac{WfLd'}{12B}$	$I=\frac{22.5WL^2}{Er}$
$L=\frac{12BI}{Wfd'}$	$L=\frac{EIr}{22.5W}$

(215.) *Cross Section rectangular, solid.*

BREAKING.	BENDING.
$W=\frac{2Bbd^2}{fL}$	$W=\frac{Ebrd^3}{270L^2}$
$b=\frac{WfL}{2Bd^2}$	$b=\frac{270WL^2}{Erd^3}$
$d=\sqrt{\frac{WfL}{2Bb}}$	$d=\sqrt[3]{\frac{270WL^2}{Ebr}}$
$L=\frac{2Bbd^2}{Wf}$	$L=\sqrt{\frac{Ebrd^3}{270W}}$

C. *Beam supported at ends, load arranged in any manner.*

Let M = maximum bending moment in foot-tons, acting anywhere along the beam; usually determined graphically. (53), (55.)

Two sets of formulæ are given for bending, one corresponding to a load concentrated at the centre, as in A; the other, to one uniformly distributed, as in B; the true value lies between the limiting values given by the two formulæ, and is to be assumed in accordance with the arrangement of the loading along the beam, approaching either limiting value, as the arrangement of the loading approaches the mode corresponding to that limiting value.

The formulæ for breaking give true values for any manner of loading.

(216.) *Cross Section not rectangular.*

BREAKING.	BENDING.	
	Load Concentrated.	Load Uniform.
$M=\frac{1.5BI}{fd'}$	$M=\frac{EIr}{144L}$	$M=\frac{EIr}{180L}$
$I=\frac{Mfd'}{1.5B}$	$I=\frac{144ML}{Er}$	$I=\frac{180ML}{Er}$
	$L=\frac{EIr}{144M}$	$L=\frac{EIr}{180M}$

(217.) *Cross Section rectangular, solid.*

BREAKING.	BENDING.	
	Load Concentrated.	Load Uniform.
$M=\frac{Bbd^2}{4f}$	$M=\frac{Ebrd^3}{1728L}$	$M=\frac{Ebrd^3}{2160L}$
$b=\frac{4Mf}{Bd^2}$	$b=\frac{1728ML}{Erd^3}$	$b=\frac{2160ML}{Erd^3}$
$d=\sqrt{\frac{4Mf}{Bb}}$	$d=\sqrt[3]{\frac{1728ML}{Ebr}}$	$d=\sqrt[3]{\frac{2160ML}{Ebr}}$
	$L=\frac{Ebrd^3}{1728M}$	$L=\frac{Ebrd^3}{2160M}$

Formulæ for Floor and Ceiling Joists, Rafters of Iron or Wood, etc.

Let c = distance between centres of joists, in inches.

Let w = weight of one square foot of floor or roof and its maximum load, in tons.

d' for wrought iron I or channel beams$=d\div 2$.
d' for angle of T bars can be found in the pocket-books. (211.)

(218.) *Cross Section not rectangular. Of Iron.*

BREAKING.	BENDING.
$w=\frac{144BI}{L^2cfd'}$	$w=\frac{EIr}{1.875cL^3}$
$c=\frac{144BI}{L^2wfd'}$	$c=\frac{EIr}{1.875wL^3}$
$I=\frac{wcfL^2d'}{144B}$	$I=\frac{1.875wcL^3}{Er}$
$L=\sqrt{\frac{144BI}{wcfd'}}$	$L=\sqrt[3]{\frac{EIr}{1.875wc}}$

(219.) *Cross Section rectangular, solid. Of Wood.*

BREAKING.	BENDING.
$w=\frac{24Bbd^2}{cfL^2}$	$w=\frac{Ebrd^3}{22.5cL^3}$
$c=\frac{24Bbd^2}{wfL^2}$	$c=\frac{Ebrd^3}{22.5wL^3}$
$b=\frac{wcfL^2}{24Bd^2}$	$b=\frac{22.5cwL^3}{Erd^3}$
$d=\sqrt{\frac{wcfL^2}{24Bb}}$	$d=\sqrt[3]{\frac{22.5cwL^3}{Ebr}}$
$L=\sqrt{\frac{24Bbd^2}{wcf}}$	$L=\sqrt[3]{\frac{Ebrd^3}{22.5cw}}$

Formulæ for Sheathing of Roofs. Wooden.

(220.) *Sheathing supported by Common Rafters.*

Let c=distance between centres of rafters, inches.

Let w=weight of sheathing, covering, and maximum load, per square foot, tons.

Let t=thickness of sheathing, in inches.

$w=\frac{3456Bt^2}{fc^2}$	$w=\frac{76.8Ert^3}{c^3}$
$c=\sqrt{\frac{3456Bt^2}{wf}}$	$c=\sqrt[3]{\frac{76.8Ert^3}{w}}$
$t=\sqrt{\frac{wc^2f}{3456B}}$	$t=\sqrt[3]{\frac{wc^3}{76.8Er}}$

(221.) *Sheathing supported by Purlines; no Rafters.*

Let L=distance between centres of purlines, in *feet*.

$$w=\frac{24Bt^2}{fL^2} \qquad w=\frac{Ert^3}{22.5L^3}$$

$$L=\sqrt{\frac{24Bt^2}{fw}} \qquad L=\sqrt[3]{\frac{Ert^3}{22.5w}}$$

$$t=\sqrt{\frac{wfL^2}{24B}} \qquad t=\sqrt[3]{\frac{22.5wL^3}{Er}}$$

For roofs, w is to be taken = weight of sheathing and covering + snow load or wind pressure, whichever may be the greater. This usually gives a small excess of strength.

(222.) *Formulæ for Common Rafters.*

Take w = weight per square foot in tons, of covering, sheathing and rafters, + maximum snow load or wind pressure (213), then apply formulæ for joists, etc. (218), (219.)

(223.) *Formulæ for Purlines.*

Let w = total weight of one square foot of roof and its maximum load in tons, as in (222).

A = area of roof actually supported by the purline, square feet.

The $W = Aw$ = total load on purline in tons.

Apply formulæ for beams supported at each end under a uniform load, substituting Aw for W. (214), (215.)

Mixed or Compound Strains.

The member or piece is acted upon by two or more kinds of strain at the same time. Consequently, its dimensions must be sufficient to enable it to safely resist all these strains.

(224.) *Beam under Transverse Strain and Shearing.*

Any beam supported at each end, and loaded, is subject to both transverse strain and shearing. The beam is generally of uniform section from end to end.

1. Determine by methods of Chapter 1 (50) to (57), the maximum bending moment, M, in foot-tons acting anywhere along the beam; find corresponding dimensions of section by (216) or (217).

2. Determine by (50) to (57) the transverse shear SH, acting at or adjacent to the location of M maximum; compute the sectional area required to safely resist this shear by (180).

3. Increase the dimensions of the section in any way as found by 1, by the area found by 2.

This value of the shear is evidently less than the maximum shear acting anywhere along the beam; if the loading be continuous, *i.e.*, not concentrated at different points, this shear will $= 0$, and the dimensions of the beam found by 1 are sufficient to resist both bending moment and shear.

(225.) *Member subject to both Longitudinal Compression and to Transverse Strain.*

This frequently occurs in the principal of a roof, especially when the common rafters are omitted, and several purlines in each panel directly support the sheathing.

1. Determine the section of the member required to resist the compression by (205) or (206), if it be of cast iron; by (207), 208) or (209), if it be of wrought iron; by (200) or (203), if of white pine; by (202) or (204), if of white oak.

Compute by (69), (70), (71) or (73), the required moment of inertia I corresponding to the section just found for compression, which call I'.

2. By formulæ of (216), compute the moment of inertia I of a section required to safely resist the transverse strain only, which call I''.

3. Design a new section, whose value for $I = I' + I''$.

(226.) *Member subject to Tension and Transverse Strain.*

This occurs in the lower chord of a roof truss, when it supports a ceiling also, etc.

1. Determine the maximum bending moment M maximum, and by (216) or (217), compute the sectional dimensions required to safely resist this strain alone.

Or, these dimensions may be directly computed by (214) or (215), if the load be uniform.

2. Determine by (174) the additional area A required to safely resist the tension alone.

3. Design a new section similar to that determined by 1, but whose area is increased by A.

(227.) *Inclined Beams Supporting Transverse Forces or Loads.*

This occurs in rafters, in principals and members of upper chords, which support purlines, etc.

The resultant of all the forces acting on the beam may be resolved by (32) into two components.

1. Parallel component, which produces compression lengthwise the member.

2. Normal component, producing transverse strain in the member.

This is then to be solved by (225).

(228.) *Wooden Keys or Joggles.*

These should have their fibres parallel to those of the timbers in which they are inserted, because they are just as strong, and the key does not then loosen by shrinkage.

The key is then subject to longitudinal shear and to crushing. Its middle plane usually coincides with the plane of the joint between the timbers.

In order to make the key equally safe against crushing and shearing, as well as to avoid crushing the timbers adjacent to the ends of the key, employ the following proportions: Keys are usually made from 2-inch plank, for convenience.

a. For White Oak Keys in White Oak Timbers.

1. Make length of key $3\frac{1}{3}$ times its depth or thickness.

2. Make least distance between keys or key and end of timber the same.

b. For White Oak Keys in White Pine Timbers.

1. Make length of key $2\frac{3}{4}$ times its depth or thickness.

2. Make least distance between two keys or key and end of timber $6\frac{1}{2}$ times its depth or thickness.

For either (*a*) or (*b*), safe resistance of key to shearing $= .09 \times$ length $\times$ breadth in inches.

(229.) *Rivets of Wrought Iron.*

A rivet may give way in either of two ways:

1. *By shearing off the rivet transversely.*

Let $Z =$ maximum safe resistance of one rivet to shear in tons.

Let $t =$ thickness of plate connected by rivet; in case the plates are of different thicknesses, then $t =$ thickness of the thinner.

Let $d =$ diameter of rivet in inches.

$Z = 2.945\ d^2 =$ maximum safe resistance in tons.

$d = .583 \sqrt{Z} =$ minimum safe diameter of rivet.

The diameter of the rivet should always be more than the thickness of the thicker plate.

2. *By crushing the edge of plate or side of rivet.*

$t = \frac{Z}{7.5d} =$ minimum thickness of plate, inches.

$d = Z \div 7.5t =$ minimum diameter of rivet, inches.

Determine d or Z for both crushing and shearing, and take the *safest* result. (See Carnegie's Pocket Book, page 135.)

The least diameter between centres of rivets should not be less than 3 diameters, if possible.

The least distance from centre of rivet to edge of plate should at least be 1¼ diameters.

(230.) *Wrought-Iron Joint Pins.*

Pins are cylindrical, and are employed for connecting the members of wrought-iron trusses.

They may fail by transverse shearing, by crushing the edge of the plate against which they bear, or by breaking transversely.

a. Shearing at a single section.

$Z = 2.945d^2 =$ maximum safe resistance to shearing.

$d = .583 \sqrt{Z} =$ minimum safe diameter of pin.

b. Crushing of pin or edge of supporting plate.

Let $t =$ thickness of plate or bar considered, in inches.

$Z = 6.25td =$ maximum safe pressure of pin against bar or plate, tons.

$d = \frac{Z}{6.25t}$ — minimum safe diameter of pin, inches.

$t = \frac{Z}{6.25d} =$ minimum thickness of plate or bar, inches.

c. Breaking of pins.

The maximum safe tensile or compressile strain in the fibres of the pin most distant from its axis may be taken $= 7.5$ tons for roofs, per square inch.

Let M' = maximum bending moment in *inch-tons*, found to act anywhere along the pin. (50) to (57.)

$d = 1.107\sqrt[3]{M'}$ = minimum safe diameter of pin.

$M' = .737d^3$ = maximum safe bending moment, inch-tons.

The three sets of formulæ must be applied in any given case, taking the *safest* result, so as to ensure the safety of the pin.

We also have the following approximate rules, which are generally safe and are more easily applied:

1. For eye-bars of rectangular section, .75 × breadth of bar = least diameter of pin.

2. For round eye-bars, 1.5 × diameter of bar = least diameter of pin.

(231.) *Bars and Eyes for Eye-bars and Pin Joints.*

Let b = breadth and t = thickness of a rectangular bar. Then b should not be greater than $6t$, or less than $4t$.

Then $.75b$ = approximate diameter of pin.

Then $.8b$ = least breadth of metal on each side of eye.

End of eye semicircular, connected with bar by long curves.

Let d = diameter of a round eye-bar.

$1.5d$ = approximate diameter of pin.

Eye to be of rectangular cross-section; width at eye on each side of pin = d, thickness = $\frac{3}{4}d$.

TABLE OF AVERAGE CO-EFFICIENTS FOR MATERIALS.*

MATERIAL.	T.	S.	S′.	C.	C′.	E.	B.
Ash, white	6.25	3.00	0.30	3.00	1.13	750.	.264
Beech, American	4.50	2.50	....	4.15	....	690.	.204
Birch, American	6 00	2.75	0.34	4.25	0.90	690.	.2[illegible]7
Cedar	4.50	.....	0.70	2.80	0.98	500.	.200
Cherry	.....	1.47	....	3.40	1.23	615.	.250
Chestnut	4.75	0.76	0.35	2.50	0.51	600.	200
Elm	5.00	1.69	0.70	4.40	....	625.	.223
Fir, New England	5 00	.... .	0.34	3.40	0.60	550.	.175
Gum	6.75	2 95	...	3.40	....	700.	.206
Hemlock	4.00	1.34	0.24	2.75	0.30	600.	.2[illegible]0
Hickory	6.00	3.00	....	4 50	1.70	650.	.340
Larch, Hackmatac	4.25	1.25	0.65	2.25	0.65	600.	.211
Locust	7.00	3.50	0.57	5.00	1.37	750.	.306
Mahogany	6 50	.....	....	3.75	1.48	720.	.250
Maple, rock	5.00	3 00	0.30	3.00	1.12	760.	.250
Oak, live	5.75	4 25	0.35	4.00	2.25	900.	.280
Oak, red	4.50	.....	0.39	3 00	....	625.	.233
Oak, white	5.50	2.15	0.40	3.50	1.38	700.	.300
Pine, Georgia	7.75	2.50	0.31	4.25	1.02	900.	.3[illegible]0
Pine, pitch	4 75	2.50	0.26	4 00	....	750.	.250
Pine, red	4 25	.....	....	3 00	....	700.	.225
Pine, white	4.00	1.25	0.25	2 50	0.40	650.	.200
Pine, yellow	5 00	2 19	0.25	2 75	0.45	700.	.200
Poplar, yellow	3 50	2.20	0.20	2 50	0.46	625.	.210
Spruce	5.00	1.67	0.25	3.00	0.40	700.	.220
Sycamore	5 75	.....	....	3.75	....	600.	.215
Walnut, black	4.50	1.81	....	3.30	1.07	750.	.293
Glass	1.43	.....	....	14 25	....	3800.	.098
Granite, average	0.29	.....	....	6.50	....	.....	.051
Limestone, average	0.68	.....	....	4 30	....	8400.	.044
Marble, average	0.35	.....	....	4.40	....	.....	.040
Sandstone, average	0.08	.....	....	2.88	....	.....	.029
Slate, average	4 00	.....	....	4.50	....	7000.	.152
Aluminum, bronze	36.45	....	....	65.25	....	5000.	....
Brass, cast	9 00	18.50	....	32 75	....	4475.	.524
Bronze	17.50	.....	....	60 00	....	5000.	1.480
Bronze, phosphor	25.00	18 50	....	.....	....	7000.	.....
Copper, cast	10.75	.....	....	47.50	....	7500.	.....
Iron, cast, average	8.00	11.00	....	48.00	....	7725.	1.025
Iron, cast, best	13.50	14.00	....	68 00	....	11500.	1.250
Iron, pig	7.00	.....	....	40.00	...	6500.	1.[illegible]00
Iron, wrought, average	25.00	22.50	....	23.50	....	13000.	1.200
Iron, wrought, best	38.00	.....	....	.....	....	14500.	1.300
Lead, cast	0 95	.....	....	3 85	....	450.	.....
Steel, cast	52 00	40.00	....	75 00	....	15000.	2.860
Steel, hard	55 00	.....	....	.....	....	13600.	2.228
Steel, mild	30.00	.....	....	.... .	....	14500.	1 466
Steel, wrought, average	52.00	34.50	....	45.00	....	14500.	2.820
Zinc, cast	1.80	.... .	....	20 00	....	6725.	.208

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*This table was carefully revised in 1887 to accord with the results of recent experiments, making the values of the co-efficients somewhat different from those used in Chapters VIII and IX.

CHAPTER VIII.

DETERMINATION OF SECTIONAL DIMENSIONS OF RAFTERS, PURLINES AND TRUSS MEMBERS.

We will here consider two examples, which will sufficiently explain the proper mode of procedure.

1. A roof principally constructed of wood.
2. A roof entirely constructed of wrought iron.

A. WOODEN ROOF.

Take, as an example, the roof already studied in problem 1, Chapter V. (117 to 125).

(233.) *Computation of Lengths of Truss Members.*

See Chapter VI. (150, 152).

Span between centres of end joints A and O (Fig. 37), 80 feet. Rise H G, 15 feet. Number of panels, 8.

1. *Principals.* Total length of principal $= \sqrt{40^2 + 15^2} =$ 42.72 feet. Then $42.72 \div 4 = 10.68$ feet $= 10$ feet $8\frac{5}{32}$ inches very nearly = joint length of members forming principals.

2. *Tie-beam.* Joint length $= 80 \div 8 = 10$ feet.

3. *Vertical Ties.*

Tie 6 7. Joint length = rise = 15 feet.

Tie 4 5. Joint length $= 3 \times 15 \div 4 = 11.25$ feet = 11 feet 3 inches.

Tie 2 3. Joint length $= 1 \times 15 \div 2 = 7.5$ feet = 7 feet 6 inches.

4. *Struts.*

Strut 1 2. This is the hypothenuse of a right-angled triangle, the altitude of B above AO being 3.75 feet. Hence, its joint length $= \sqrt{3.75^2 + 10^2} = 10.68$ feet as for principals.

Strut 3 4. Joint length $= \sqrt{7.5^2 + 10^2} = 12.5$ feet.

Strut 5 6. Joint length $= \sqrt{11.25^2 + 10^2} = 15.052$ feet = 15 feet $0\frac{5}{8}$ inches very nearly.

(234.) *Common Rafters.* (219), (227.)

These are subject to a transverse strain, like floor joists, and also to a longitudinal compression like a strut, thus making it very difficult to accurately determine the joint effect of the two kinds of strain, and the corresponding depth of the rafters.

For practical purposes it is usually sufficient to consider the rafter as subject to transverse strain only, caused by a load per square foot of the roof surface, equal to the permanent weight of this surface, plus the snow load or wind pressure, whichever of the last two may be greatest.

Then compute the least safe depth of the rafter by the following formulæ, taking the larger of the two results. Both formulæ must be applied, because the rafter may fail either by breaking, or by bending too much. (219.)

1. For Resistance to Breaking.

$$d = \sqrt{\frac{wcfL^2}{24\,Bb}} = \text{depth in inches.}$$

2. For Resistance to Bending.

$$d = \sqrt[3]{\frac{22.5\,wc\,L^3}{Ebr}} = \text{depth in inches.}$$

In these formulæ (219), (211), (117), (118):

w = weight of 1 square foot of roof surface and its greatest load, either snow or wind pressure, in net tons, = 5.5 + 23.6 lbs. = .01455 tons.

c = distance between centres of rafters in inches = 24.

f = factor of safety, = 5.

L = length of rafter between centres of purlines in feet = 10.68.

b = breadth or thickness of rafter, usually 2 inches = 2.

d = depth of rafter in inches.

r = maximum permissible deflection or sag of rafter in inches per foot of its length, usually .03.

B = co-efficient for material of rafter, for breaking = .23.

E = co-efficient for bending for the material = 750. (232.)

Substituting these values in the formulæ and reducing:

1. Breaking.

$$d = \sqrt{\frac{.01455 \times 24 \times 5 \times 10.68^2}{24 \times .23 \times 2}} = 4.25 \text{ inches.}$$

2. Bending.

$$d = \sqrt[3]{\frac{22.5 \times .01455 \times 24 \times 10.68^3}{750 \times 2 \times .03}} = 5.97 \text{ inches.}$$

The rafters should, therefore, be 2 × 6 inches. These computations are most readily performed by the aid of a good 5-place table of logarithms (Newcomb's).

(235.) 2. *Purlines.*

The purlines are only subject to transverse strain, but their sides not being vertical, their resistance to a load acting vertically is less, for rectangular purlines of ordinary length, than if their sides were vertical, like those of girders. But in roofs of ordinary inclination, the wind pressure is greater than the snow load per square foot of roof surface, and the resultant of the permanent load and wind pressure being nearly parallel to the middle plane of the purline, the error here noted is principally eliminated.

The breadth of the purline is generally assumed, and its depth (at right angles to the roof surface) is then computed by the following formulæ, taking the greater result. (215), (224.)

1. Resistance to Breaking.

$$d = \sqrt{\frac{AwfL}{2Bb}} = \text{depth in inches.}$$

2. Resistance to Bending.

$$d = \sqrt[3]{\frac{270\ Aw\ L^3}{Ebr}} = \text{depth in inches.}$$

In these formulæ:

A = area of roof surface in square feet actually supported by one purline, = distance between centres of trusses × distance between centres of purlines, here = 10.68 × 16 = 171 square feet nearly. (117.)

L = distance between centres of trusses, here = 16 feet.

b = breadth of purline, say 8 inches.

The other letters have the same signification and value as in (234).

Substituting these values and reducing:

1. Breaking.

$$d = \sqrt{\frac{171 \times .01455 \times 5 \times 16}{2 \times .23 \times 8}} = 7.35 \text{ inches.}$$

2. Bending.

$$d = \sqrt[3]{\frac{270 \times 171 \times .01455 \times 16^2}{750. \times 8 \times .03}} = 9.85 \text{ inches.}$$

Hence the purline should be 8 × 10 deep.

(236.) 3. *Principals and Struts.*

These are here only subject to longitudinal compression. The most convenient formula for square posts or struts of white pine, under compression, is that of Col. C. Shaler Smith, put in the following form:

$$W = \frac{2.5\,s^2}{f\left(1 + .576\frac{L^2}{s^2}\right)} = \text{maximum safe compression or load}$$

on post in net tons. (200.)

Also, s = side of square post in inches.

L = length of post in feet.

f = factor of safety, usually 5.

This formula may also take the following form, when required for computing the sides of the strut, the load or compression W being given.

$$s = \sqrt{W \div \sqrt{1.152\,WL^2 + W^2}} = \text{side in inches.}$$

(237.) *Graphical Table.* (202.)

Since the use of these formulæ is rather tedious in practice, they have been embodied in a graphical table, devised by the writer and used in his classes for several years, though now published for the first time. (See Table 3.)

The figures at top of table are safe loads or compressile strains in net tons, with a factor of safety of 5; those at the left side are lengths of posts or struts in feet; while the figures at upper ends of curved lines are the sides of posts in inches.

(238.) *To find Side of square Post or Strut.*

Its length and maximum load or compression being given. Look for intersection of a horizontal through the given length, at left, with a vertical through the given compression, at top; if this intersection falls on a curve, the corresponding number at the end of this curve will be the required side of post or strut; if between two curves, estimate value of side to nearest ⅛ inch, according to the relative normal distance of the intersection from the two curves, between which it lies. (203.)

For example, let $W = 38$ tons; $L = 16$ feet; required s. A horizontal through 16 and vertical through 38 intersect midway between the two curves 12 and 13. Hence, the post must be at least 12½ inches square.

(239.) *To find sectional Dimensions of rectangular Post.* (201.) Its length L and load W being given.

Let a=least side of post in inches.

Let b=greater side of post in inches.

There may be two different cases.

A. Given a, W and L; required b.

On the graphical table find intersection of a horizontal through L with the curve representing a; a vertical through this point gives at the top the safe strength of a square post, whose side is a; call this W^1.

Then $b = a\frac{W}{W^1}$ = required size of post.

B. Given b, W and L; required a.

Assume a value for a and proceed as before to determine the corresponding value for b; should this differ materially from the given value of b, assume a new value for a and proceed as before, continuing the approximation until a value is obtained for b, equal to or slightly smaller than the given one. (201.)

For example: given, W 40 tons, L 16 feet, $a = 12$ inches; required b. We find W^1 to be 35.5 tons, b to be 13.52 inches. The post should be 12×14 inches.

Also, let W be 35 tons, L 18 feet, b 16 inches; required a. Making a=10 inches, b=20.0 inches; or if a be 12 inches, b= 13.4 inches. The post should, therefore, be 12×14 inches.

(240.) *Application to Problem* 1. (See Strain Sheet.)

Principal. The greatest compression found in any member of the principal=26.10 tons, acting on X1. Length of X1= 10.68 feet. By (238), we find that this member must be at least 9½ inches square. If it be made 8 inches wide, its depth by (239) must be 13.05 inches, so that the principal must either be 8×14 deep or 10×10, the latter being preferable, because stiffer and cheaper.

For convenience in construction, the entire principal is usually made of the same dimensions throughout, so that it is unnecessary to find those of X2, X4 and X6, because they are the same as for X1. The tie-beam always has the same horizontal breadth as the principal, for sake of appearance, and this is usually true of the struts also, which look better if their edges are flush with those of the upper and lower chords.

Strut 1 2. Maximum compression, 4.56 tons; length, 10.68 feet; breadth, 10 inches, same as for principal. By (239) we find this strut must be 6×10 inches.

Strut 3 4. Maximum compression, 5.84 tons; length, 12.5 feet (measured on truss diagram, Fig. 21); breadth, 10 inches. By (239) this must also be 6×10 inches.

Strut 5 6. Compression, 6.40 tons; length, 15.10 feet. A timber 6×10 would not be quite sufficient, and it should be 8×10.

(241.) 4. *Tie-Beam.*

The breadth of tie-beam always being that of the principals, its depth may be found by the following formula: (175).

$d=\frac{Zf}{Tb}$=least safe depth in inches.

Let Z=maximum tensile strain on tie-beam in tons.

f=factor of safety=10 in this case, to allow for splicing tie-beam, cutting fibres by indents, bolt-holes, etc. (173.)

$T=4$ tons for white pine. (232.)

b=breadth of tie-beam in inches, here 10 inches.

Here Z=strain on Y1=24.42 tons.

Substituting values and reducing: $d=\frac{24.42\times 10}{4\times 10}=6.11$ inches.

A timber 6×10 would do, if of good quality and carefully spliced, which is best done by building it up of five 2×6 planks, set edgewise and firmly spiked and bolted together, breaking joints. For convenience in construction, the tie-beam should have the same dimensions throughout, although the strains on Y3 and Y5 are smaller.

(242.) 5. *Wrought-Iron Rods.* (178), (179.)

The diameters of rods and dimensions of heads, nuts, washers, etc., may be most conveniently found by means of Table 1 or 2.*

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The origin of the formulæ on which the tables are based, with the mode of their construction, is fully explained in an essay by the writer, published in No. 401 of the *American Architect* for September 1, 1883.

Screw threads are usually cut on the ends of bolts and short rods, so as to reduce their solid diameter, and their tensile strength as well, which will only be that due to the uncut metal between the bases of the threads. Longer rods, especially those used in roof and bridge trusses, usually have their ends enlarged before the threads are cut, so as to make the screw ends as strong as the rod itself, thus producing a considerable saving in material and cost.

Rods with ends not enlarged may have a head forged on one end, or nuts on both ends, which is often more convenient in setting up the truss, and avoids the risk of a bad weld. Nuts are always placed on both ends of rods having enlarged ends.

The number of threads per inch, and the proportions of heads and nuts, are in accordance with the standard of the Franklin Institute, more generally adopted in the United States than any other, the different dimensions of some manufacturers being also given.

Washers are usually made of cast iron, are circular, square or rectangular, and their thickness should always be the same as that of the corresponding nuts and rod-ends.

(243.) *Mode of using the Tables.* (178), (179.)

A. To determine dimensions for a rod required to safely resist a given tensile strain: Look in column 3 of the table for the given strain, or the next larger one; on the same horizontal line will be found the diameter of the rod in the first column, and the dimensions of ends, nuts and washers in the proper columns.

B. To determine the maximum safe tensile strength of a rod: Look for the given diameter in the first column; its maximum safe tensile strength will be found on the same line in the third column.

(244.) *Application to Problem* 1.

We will employ rods with enlarged ends, because cheaper. Table 2.

Tie 2 3. Maximum strain = 1.67 tons. (Strain sheet.) (125.)

Looking in the table we do not find this strain, but the next larger one is 1.86 tons, corresponding to a rod $\frac{11}{16}$ inch diameter. The diameter of enlarged end $= \frac{57}{64}$ inch; nuts, $\frac{57}{64}$ thick and $1\frac{15}{32}$ square; washers, $= \frac{57}{64}$ thick and $2\frac{55}{64}$ diameter, as circular washers are generally used, and they press on white pine timber.

Tie 4 5. Strain, 3.17 tons. Similarly, diameter rod $\frac{15}{16}$ inch; of enlarged end, $1\frac{3}{16}$ inches; nuts, $1\frac{3}{16} \times 1\frac{29}{32}$ inches; washers, $1\frac{3}{16} \times 3\frac{57}{64}$ inches diameter.

Tie 6 7. Strain, 7.87 tons. Rod, $1\frac{1}{2}$ inches diameter; end, $1\frac{53}{64}$ inches; nuts, $1\frac{53}{64} \times 2\frac{7}{8}$ inches; washers, $1\frac{53}{64} \times 6\frac{17}{64}$ diameter, or $5\frac{9}{16}$ square.

Since this rod is rather large, it would usually be better to replace it by two rods. Then $7.87 \div 2 = 3.94$ tons strain on each, assuming them to be of equal diameter, and equally screwed up. Diameter of rods, 1 inch, with ends $1\frac{1}{4}$ inches; nuts, $1\frac{1}{4} \times 2$ inches; washers, $1\frac{1}{4} \times 4\frac{9}{94}$ inches diameter.

It will sometimes be convenient to substitute a single rectangular plate for the round washers of a group of rods. The area of this plate must equal the combined areas of all the washers. Since $3\frac{43}{64} =$ side of square washer for 1-inch rod, $2 \times 3\frac{43}{64} \times 3\frac{43}{64} = 27$ square inches nearly $=$ area of equivalent rectangular plate. If this plate be made 8 inches long, its width should be $3\frac{3}{8}$ inches and it should be $1\frac{1}{4}$ thick.

B. WROUGHT-IRON ROOF.

Take, as an example, the roof already examined in Problem 5, Chapter V (142) to (146). (150), (151), (159.)

(245.) *Computation of Lengths of Truss Members.*

In this problem, $s = 80$ feet; $n = 7$; $r' = 16$ feet; $r'' = 3$ feet.

$\frac{2r'}{s} = \frac{2 \times 16 \times 7}{6 \times 80} = \tan i' = \tan 25^\circ\ 1'\ 1'' =$ inclination of upper chord.

$\frac{s}{n \cos i'} = \frac{80}{7 \cos 25^\circ 1' 1''} \doteq 12.612 \text{ feet} = 12 \text{ feet } 7\frac{11}{32} \text{ inches} =$ length of any panel of upper chord, excepting the middle one.

$\frac{s}{n} = \frac{80}{7} = 11.429$ feet $= 11$ feet $5\frac{9}{64}$ inches $=$ length of middle panel of upper chord.

$\frac{12.612}{2} = 6.306$ feet $= 6$ feet $3\frac{43}{64}$ inches $=$ length of member X12.

$\frac{11.429}{2} = 5.714$ feet $= 5$ feet $8\frac{37}{64}$ ins. $=$ length of member 11 12.

$\frac{2r''}{s} = \frac{2 \times 3}{80} = \tan \frac{1}{2} 1'' = \tan 4^\circ\ 17'\ 21''$.

Hence, $i'' = 8^\circ\ 34'\ 42''$ = angle of inclination of tangent at end of lower chord, at A or R.

$\frac{s}{2 \sin i''} = \frac{80}{2 \sin 8^\circ\ 34'\ 42''} = R'' = 268.166$ feet $=$ radius of curvature of lower chord.

Heights of joints of chords above a horizontal line drawn through joints A and R. (150.)

Joint F, height $= 16$ feet.

Joint D, height $= \frac{2 \times 16}{3} = 10.667$ feet.

Joint B, height $= \frac{16}{3} = 5.333$ feet.

Joint G, height $= 3 - 268.166 + \sqrt{268.166^2 - \left(\frac{.5 \times 80}{7}\right)^2} =$ 2.939 feet. (159.)

Joint E, height $= 3 - 268.166 + \sqrt{268.166^2 - \left(\frac{1.5 \times 80}{7}\right)^2} =$ 2.452 feet.

Joint C, height $= 3 - 268.166 + \sqrt{268.166^2 - \left(\frac{2.5 \times 80}{7}\right)^2} =$ 1.234 feet.

Lengths of Verticals.

Vertical 1 2 $= 5.333 - 1.234 = 4.099$ feet $= 4$ feet $1\frac{3}{16}$ inches.
Vertical 3 4 $= 10.667 - 2.452 = 8.215 = 8$ feet $2\frac{37}{64}$ inches.
Vertical 5 6 $= 16. - 2.939 = 13.061 = 13$ feet $0\frac{47}{64}$ inches.
Vertical X 11 $= 4$ feet.
Vertical 12 13 $= \frac{5.333}{2} = 2.667 = 2$ feet 8 inches.

Lengths of Diagonals. (159.)

Diagonal 2 3 $= \sqrt{11.429^2+(10.667-1.234)^2}=14.819$ feet$=14$ feet $9\frac{53}{64}$ inches.

Diagonal 4 5 $= \sqrt{11.429^2+(16.-2.452)^2}=17.725$ feet $=17$ feet $8\frac{45}{64}$ inches.

Diagonal 6 7 $= \sqrt{11.429^2+(16.-2.939)^2}=17.355$ feet$=17$ feet $4\frac{17}{64}$ inches.

Diagonal 10 11$= \sqrt{4^2+5.714^2}=6.975$ feet$=6$ feet $11\frac{45}{64}$ inches.

Determination of Sectional Dimensions.

(246.) *Purlines.*

1. Purlines of Main Roof. (223), (214.)

There are to be 3 purlines to each panel or section area. Hence, $A=67\frac{1}{4}$ square feet; $w=5.2+28.3$ lbs.$=.01675$ ton; $f=5$; $L=16$ feet; $B=1.2$; $E=12442$; $r=.03$. (232.)

Breaking.

$$I=\frac{AwfLd'}{12B}=\frac{67.25\times.01675\times5\times d'}{12\times1.2}=6.247d'.$$

Bending.

$$I=\frac{22.5AwL^2}{Er}=\frac{22.5\times67.25\times.01675\times16^2}{12442\times.03}=17.39.$$

By Table of Properties of I-beams, page 62, Carnegie Bros.' Pocket Book, we find the lightest I-beam, whose value of I or moment of inertia of its cross section is greater than 17.39, is a 6-inch 13.5-lb. beam, for which $I=24.5$ in column 8. Then $d'=d\div2=3.$, and $6.247\ d'=6.247\times3=18.731$. This last value of I is required for resistance to breaking, but being smaller than that of the assumed beam, this beam will be safe against both breaking and bending. It is also the lightest that will do, and therefore the most economical.

2. Purlines of Small Roof.

There are to be three purlines on each side; the middle one supporting $50\frac{5}{8}$ square feet of roof $=A$. The other values are the same as in the last case.

Breaking.

$$I=\frac{.843\times5\times16\times d'}{12\times1.2}=4.683d'.$$

Bending.

$$I=\frac{22.5\times.843\times16^2}{12442\times.03}=13.01.$$

In the same way we find that a 6-inch 10-lb. channel will do, but not a 5-inch 10-lb. I-beam; so that it will be most economical to use the channel. The other two purlines only support half as much roof area, but it will be necessary to make them of the same depth and section as the middle one.

(247.) *Upper Chord.* (225.)

Each member of this chord is to be composed of two channels, laced together by diagonal bars, resisting longitudinal compression, and also supporting two purlines, each of which causes a concentrated transverse load of 1.127 tons, located at one-third the length of the member from each end.

1. *Resistance to Transverse Strain only.*

(248.) First find M maximum = maximum bending moment acting anywhere along the member. (54.)

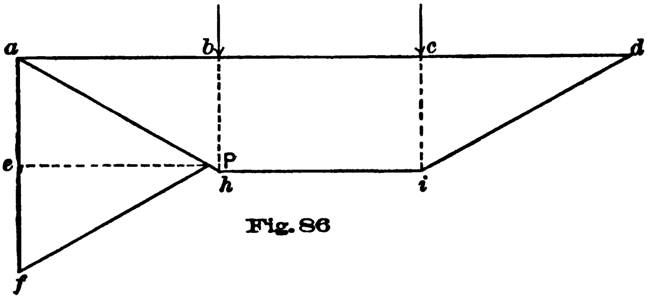


Fig. 86

In Fig. 86 make *ad*=12.61 feet, and divide in three equal parts at *b* and *c*. Lay off load line *af* = 2×1.127 tons; assume pole P, making pole distance P*e* = say 2 tons; draw equilibrium polygon *ahid*. The maximum intercept (uniform between *b* and *c*, since *hi* is parallel to *ad*) = 2.35 feet, and M maximum = 2× 2.35 = 4.70 foot-tons.

As the load is neither concentrated at the middle, nor uniformly distributed among the member, apply formulæ of (216). Values as before, except that the value of M maximum is used in place of those of A and w.

Breaking.

$$I=\frac{Mfd'}{1.5B}=\frac{4.70\times5\times d'}{1.5\times1.2}=13.05d'.$$

Bending.—Load concentrated at centre.

$$I=\frac{144ML}{Er}=\frac{144\times4.70\times12.61}{12442\times.03}=22.865.$$

Bending.—Load uniformly distributed.

$$I=\frac{180ML}{Er}=\frac{180\times4.70\times12.61}{12442\times.03}=28.581.$$

The true value of I, required to safely resist the bending, corresponding to the actual arrangement of the loading, evidently lies between the limits 22.865 and 28.581. It is readily obtained by the following approximate method.

If equal pole distances of 2 tons be taken, and this total loading be considered as actually arranged, as concentrated at the centre, and as if uniformly distributed, the values of the three corresponding intercepts will be as follows:

2.35 feet for actual arrangement of loading.
3.50 feet for load concentrated at centre.
1.75 feet for load uniformly distributed.

Then 1.75 = difference between the two last, and 3.50—2.35 = 1.15 = difference between the first two.

The difference between the values of I corresponding to the last two intercepts = 28.581—22.865 = 5.716.

Then 1.75 : 1.15 :: 5.716 : 3.765.

And 22.865 + 3.765 = 26.63 = approximate actual value of I required to resist bending, under the actual arrangement of the loading. The value for breaking is correct, without any interpolation or correction.

For shearing (180), $A=\frac{Zf}{S}=\frac{1.127\times5}{22.5}=.25$ square inches, which is so small that it may safely be neglected.

(249.) *Resistance to both Compression and Transverse Strain.*

Members X1, X2. Try two 9-inch 18-lb. channels.

By Carnegie, page 65, for these channels: $A = 10.80$ square inches; $I = 129.6$; $Rg = 3.46$. Then $\frac{L}{Rg} = \frac{12.61}{3.46} = 3.65$.

By Table 4 (209), a vertical through 3.65 at top of table intersects the curve for hinged struts (pin-jointed at A), on a horizontal through 5.35 tons = safe compression per square inch of section. The maximum compression on the member being 30.66 tons (146), we have $30.66 \div 5.35 = 5.72$ square inches = sectional area required to safely resist compression alone.

Then $10.80 : 129.6 :: 5.72 : 68.5 = I'$ = that part of the total value of I for the entire section employed in resisting compression alone.

Also I'' = that value of I required for resisting transverse strain alone, $= 13.05 \times 4.5 = 59.00$, for breaking, this being greater than the value 26.63 previously found for bending. (248.)

$I' + I'' = 68.5 + 59. = 117.90$ = total value of I required for the section of the member; this being less than the total value for two 9-inch 18-lb. channels, which = 129.6, these channels would be amply sufficient. (225.)

If the sum of I' and I'' is found to exceed, or to be materially smaller than I for the two channels, it would be necessary to assume some other size of channels until one is found which will be suitable.

(250.) *Minimum Distance between the Channels.*

Using two 9-inch 18-lb. channels, by (209), we find that the minimum distance between the centres of gravity of the channels must not be less than $2 \times 3.46 = 6.92$ inches. According to Table, page 65 of Carnegie, column 15, the centre of gravity of the channel is .68 inch from the outside of the web. Hence, $6.92 - 2 \times .68 = 5.56$ inches = minimum distance in clear between the webs of the channels. We will make this $5\frac{3}{4}$ inches. If the total thickness of all the eye ends at any joint of the upper chord should exceed this distance, it must be increased so as to receive them between the channels.

According to the Pocket Book of the Pencoyd Iron Company

(Wrought Iron and Steel in Construction), page 144, the maximum distance between lacing points should not exceed about 2 feet. The actual distance would only be about 17 inches, and this is therefore sufficient.

Member X4. Maximum compression, 24.10 tons.

It will be necessary to employ two 9-inch 18-lb. channels, as this will be found more economical than to use 8-inch channels of weights varying according to the compression on each member.

Member X11. Maximum compression, 1.30 tons.

Try a 4-inch 8-lb. I-beam. $L \div Rg = 4.00 \div 1.61 = 2.49$. By Table 4, 6.22 tons = safe compression per square inch of section. Then $1.30 \div 6.22 = .209$ square inch = area of section required. The actual area of this beam is 2.4 square inches, so that the beam is much stronger than necessary, though it will be best to use this for the sake of appearance, and because it will probably be required to aid in resisting the horizontal pressure of the wind against the windows.

Member X12. Maximum compression, 1.4 tons.

This also supports a transverse pressure of .843 tons at its centre. By (212) we obtain:

$$I = \frac{WLfd'}{6B} = \frac{.843 \times 6.306 \times 5 \times d'}{6 \times 1.2} = 3.688d'.$$

$$I = \frac{36WL^2}{Er} = \frac{36 \times 6.306^2 \times .843}{12442 \times .03} = 3.227.$$

Try a 5-inch 10-lb. I-beam. $L \div Rg = 6.306 \div 2.03 = 3.10$. By Table 4, 5.68 tons = safe compression per square inch. $1.40 \div 5.68 = .246$ square inch = area required for compression. Then $3.00 : .246 :: 12.3 : 1.01 = I'$ for compression alone.

For breaking, $I'' = 3.688 \times 2.5 = 9.22$. Then $I' + I'' = 1.01 + 9.22 = 10.23$ = total value of I required for the section. The actual value of I for this beam being 12.30, it is somewhat stronger than necessary, but a 4-inch 10-lb. I-beam would be too weak.

Member 6 10. Compression alone, 16.57 tons.

Try two 6-inch 7.5-lb. channels. $L \div Rg = 11.43 \div 2.67 = 4.29$. By Table 4, 4.95 tons = safe compression per square inch of section. 16.57 ÷ 4.95 = 3.35 square inches = area required. The area of the two channels being 4.50 square inches, these bars are a little larger than necessary, but 5-inch 7.5-lb. bars would be too small.

(251.) *Verticals an Members of Small Truss.*

The struts 1 2, 3 4, and 5 6, are frequently each constructed of two Ts riveted together; but in this case it will be more economical to use a star bar for each, cutting away the side wings at each end, and forming an eye to slip on the joint pin. It would not be sufficient to merely allow an open notch in the ends of the strut to rest against the joint pin, for a cyclone in the vicinity might lift the roof surface temporarily, allowing the strut to fall out, thus causing the fall of the truss when closed eyes would have made it secure. Or the ends of the side wings may be welded down to form the eyes.

Member 1 2. Maximum compression, 4.05 tons.

Try a 2½ × 2½ star. L ÷ R*g* = 3.87 ÷ .52 = 7.43.

By Table 4, 3.42 tons = safe compression per square inch. Then 4.05 ÷ 3.42 = 1.185 square inch sectional area required. The actual area being 1.65, this bar will do. The 2×2 star would be too weak.

Member 3 4. Maximum compression, 6.62 tons.

In the same way we find a 4×4 star to be required for this member.

Member 5 6. Maximum compression, 1.36 tons; maximum tension, 0.95 tons.

A 3½×3½ star will be sufficient to resist the compression and the tension also. (174.)

Member 12 13. Maximum compression, .22 ton.

As this strain is quite small, it will probably be most economical to use two round rods for this member, these being most readily connected with the other members at the joints.

The value of R*g* for the section of a round rod = *d* ÷ 4. (73).

Try two ⅝ rods. L ÷ R*g* = 4×2.70 ÷ .625 = 17.3. By Table 4, .80 ton = maximum safe compression per square inch of section. .22 ÷ .80 = .275 square inches = sectional area of both rods. The actual area being .614 square inches, these rods will do, though two ½-inch rods would be too weak.

Member 11 12. Maximum compression, 0.55 ton; maximum tension, 1.30 tons.

In the same way two 1-inch rods are found to be sufficient for this member, safely resisting both compression and tension; its tensile strength being found by Table 2, as these rods would be as strong as those with enlarged ends.

Member 11 10. Maximum compression, 1.06 tons; tension, .65.

This is also composed of two rods, which are found to be 1⅛ inch in diameter.

(252.) *Lower Chord.*

Each member is composed of a pair of equal, straight, round rods of wrought iron, with properly formed eye-ends, so that the strength of these rods would equal that of similar rods with enlarged ends; their dimensions may then be found by Table 2.

Member Y1. Maximum tension 29.17 tons.

By Table 2, two 2-inch rods are required.

Member Y3. Tension 21.94 tons. Two 1¾ rods.

Member Y5. Tension 16.58 tons. Two 1½ rods.

Member Y7. Tension 16.57 tons. Two 1½ rods.

(253.) *Diagonals.*

These are also composed of pairs of equal round rods with eye-ends. If it be desired to make their lengths adjustable, this is best done by means of sleeve-nuts, properly enlarging the ends of the rod in the sleeve-nut, according to Table 2.

Member 2 3. Tension 9.94 tons. Two 1⅛ rods.

Member 4 5. Tension 10.83 tons. Two 1¼ rods.

Member 6 7. Tension 3.00 tons. Two ⅝ rods.

CHAPTER IX.

DETAIL DRAWINGS OF TRUSS JOINTS.

(254.) In practice, these should be drawn full size, or at as large a scale as possible, and each joint may be drawn on a separate piece of paper for convenience.

RULES.

(255.) 1. Choose any point on the paper to represent the intersection of the centre lines of those members meeting at the joint, and draw through it a line parallel to the centre line of each member, as shown in the truss diagram, Fig. 37 or 51.

2. If any member be curved, draw this centre line parallel to its tangent at the joint considered.

3. Lay off half the width of each member, in a vertical plane, on each side of its centre line, parallel to which draw its sides.

4. Then form the joint as indicated in the following applications, computing its dimensions where necessary, so as to be certain of its safety, and so that its different parts may be equally strong as far as possible.

A. A WOODEN ROOF.

(256.) *Application to Problem* 1. (117.)

Assume the point *a*, Fig. 87, and draw the centre lines *ab* and *ac* parallel to AB and AC of Fig. 37; lay off 5 inches on each side of *ab*, and 3 inches on each side of *ac*, and draw the top and bottom lines of the principal and tie-beam. It will be best to make the top of tie-beam the joint plane, substituting a white oak key for the usual indents, because it is equally strong and more easily and accurately fitted. The fibres of the key should be parallel to those of tie-beam, so as to avoid loosening by shrinkage, and it may be tapered so as to be driven tightly after the bolts are in place. A key 2 inches thick should be nearly 6 inches long, parallel to tie-beam. (228.)

Suppose that the wall is 16 inches thick, and that the end of tie-beam may be flush with its face, being covered by the cornice. Draw the lines of the wall, placing its centre under *a*. The toe of principal may be clipped at about 2 inches deep, and the centre of key should be placed about midway between the points *e* and *f*.

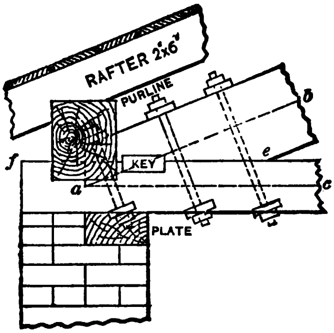


Fig. 87, Joint A.

The key may fail in either of three ways: 1st, by shearing the key along the plane of the joint; 2d, by crushing the indent at end of key; 3d, by shearing off the indent along the lower plane of the key.

1.—*Resistance of key to shearing.* (185.)

Let a = length of key in inches, here $5\frac{1}{2}$.

b = breadth of key = 10 inches.

f = factor of safety = 5.

S^1 = resistance of white oak to shearing lengthwise the fibres, per square inch, = .45 ton.

Then $\frac{abS^1}{f} = \frac{5.5 \times 10 \times .45}{5} = 4.95$ tons = safe resistance of key.

2.—*Resistance of indent to crushing.* (189.)

Let a = depth of indent in inches, = 1 inch.

b = breadth of indent = 10 inches.

f = factor of safety = 5.

C = resistance of white pine to crushing endwise per square inch, = 2.5 tons.

Then $\frac{abC}{f} = \frac{1 \times 10 \times 2.5}{5} = 5$ tons, = resistance of indent.

3.—*Resistance of indent to shearing.* (185.)

Let $a =$ distance from key to end of tie-beam, $= 13$ inches.

$b =$ breadth of tie-beam, $= 10$ inches.

$f = 5$, as before.

$S^1 =$ resistance of white pine to shearing lengthwise, per square inch, $= .20$ ton.

Then $\dfrac{abS^1}{f} = \dfrac{13 \times 10 \times .20}{5} = 5.20$ tons $=$ resistance of indent to shearing.

The least of these three values will, therefore, be the greatest safe resistance of the key, $= 4.95$ tons.

The maximum shear in the plane of the joint, or tendency of the foot of the principal to slide on top or tie-beam $=$ maximum strain in Y1 $= 24.42$ tons. Hence, $24.42 - 4.95 = 19.47$ tons, $=$ the remaining shear that must be resisted by bolts. Assuming that 6 bolts are to be employed, $19.47 \div 6 = 3.25$ tons $=$ shear on each bolt.

Let $d =$ required diameter of a bolt.

$W =$ maximum shear on one bolt, $= 3.25$ tons.

$f =$ factor of safety $= 5$.

$S =$ resistance of wrought-iron to shearing, $= 22.5$ tons per square inch.

Then $d = 1\frac{1}{8}\sqrt{\dfrac{Wf}{S}} = 1\frac{1}{8}\sqrt{\dfrac{3.25 \times 5}{22.5}} = .956$ inch, say 1 inch. (183.)

The nuts and washers for these bolts should have dimensions similar to those for bolts with ends not enlarged; or, the nuts should be $1 \times 1\frac{5}{8}$ inches; washers, $1 \times 3\frac{29}{64}$ inches diameter. The bolts should be so arranged as to avoid any danger of shearing the wood left between them, and one inch space should be left between key and nearest bolts; the bolts should be perpendicular to the principal, not to tie-beam, so as to draw tighter if the joint slips any, and the washers require to be sunk into tie-beam to get a good bearing, as shown. The purline is usually placed as shown, notched on truss, but also supported by the wall.

There might be danger that the under side of the tie-beam would be crushed by its pressure on the wall.

Let $Z =$ maximum pressure of truss on one wall, $= 9.174$ tons (measured on strain diagrams, Figs. 38 and 39).

k = factor of safety crushing across fibres, $= 2.5$.

b = breadth of beam in inches, $= 10$.

C^1 = resistance of white pine to crushing across fibres, $= 1.25$ tons per square inch.

Then $\frac{Wk}{bC^1} = \frac{9.174 \times 2.5}{10 \times 1.25} = 5.74$ inches = length of end-bearing of truss on wall. The actual end-bearing being 16 inches, this danger does not exist. (193.)

Good brick masonry will safely resist a pressure of 8 tons per square foot; hence, $9.174 \div 8 = 1.147$ square feet = bearing area of tie-beam on wall to avoid crushing it. The actual area = 1.111 square feet, which might do, though it would be safer to place the ends of the trusses on stone blocks 12 × 16, 8 or 10 inches thick, built into the walls.

2.—*Joint B. Fig.* 88.

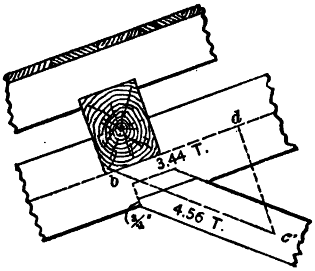


Fig. 88, Joint B.

Draw centre lines bd and bc parallel to BD and BC of truss diagram, as before, and draw top and bottom lines of principal and strut, the latter being 6 inches deep. It is best to indent the strut into the under side of principal sufficiently to resist its sliding, then fastening it in place by spikes, by a pinned or a stub tenon, as shown.

On bc make $bc = 4.56$ tons (maximum compression on strut AB) at any convenient scale. Let fall the perpendicular cd on bd, and measure bd, which represents the tendency of the strut to slide along the under side of principal, or the pressure against the indent. This pressure = 3.44 tons.

Let P = this pressure against indent, always acting parallel to plane of joint. (In case the joint be not parallel to centre line of principal, draw through *b* a parallel, and through *c* a perpendicular, to the line of the joint.)

f = factor of safety, = 5.

b = breadth of strut at right angles to plane of drawing.

C = resistance of white pine to crushing endwise, per square inch, = 2.5 tons.

Then depth of indent $= \frac{Pf}{bC} = \frac{3.44 \times 5}{10 \times 2.5} = .69$ inches, say $\frac{3}{4}$ inch. (189.)

The centre of the purline is usually placed at the intersection of a vertical through *b*, with top of principal, as shown.

3.—*Joint C. Fig.* 89.

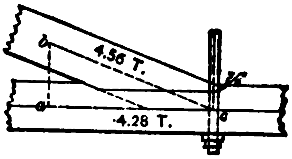


Fig. 89, Joint C.

The depth of timbers are laid off as before. Make $bc = 4.56$ tons, and let fall vertical ba, finding $ac = 4.28$ tons = P.

Then depth of indent $= \frac{4.28 \times 5}{10 \times 2.5} = .86$ inch, say $\frac{7}{8}$ inch.

Half diameter of rod is laid off on each side of a vertical through *c*, and the drawing is easily completed.

4.—*Joint D. Fig.* 90.

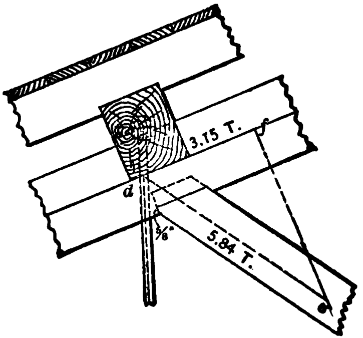


Fig. 90, Joint D.

Making $df = 5.84$ tons, $P = 3.15$ tons, and depth of indent $= \frac{5}{8}$ inch.

5.—*Joint E.* *Fig.* 91.

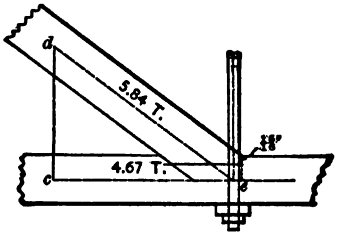


Fig. 91, Joint E.

P = 4.67 tons; indent = $\frac{15}{16}$ inch.

6.—*Joint F.* *Fig.* 92.

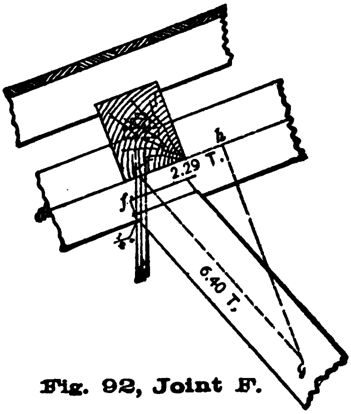


Fig. 92, Joint F.

P = 2.29 tons; indent $\frac{1}{2}$ inch.

7.—*Joint G.* *Fig.* 93.

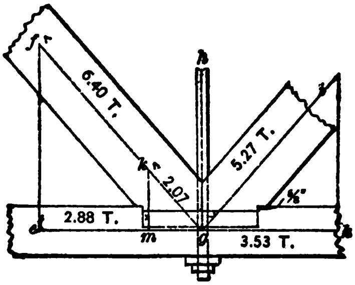


Fig. 93, Joint G.

In Fig. 93 make fg = windward strain on FG = 6.40 tons, and kg = leeward strain, = 2.07 tons; let fall verticals fe and *km*; then *em* = 2.88 tons = maximum difference of the horizontal pressures of lower ends of struts against each other, = tendency to slide at the joint G.

Also lay off on *gi* 5.27 tons, = maximum P and S compression on FG; drop vertical *ik*, finding *gk* to be 3.53 tons. This must be taken because greater than 2.88 tons, and the corresponding depth of indent = $\frac{3}{4}$ inch.

8.—*Joint H. Fig.* 94.

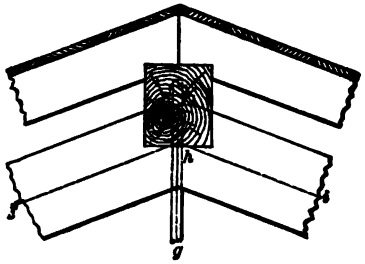


Fig. 94, Joint H.

The principals abut against each other, and slipping may be prevented by dowels of wood or iron, fitting holes bored in ends of timbers. The top is clipped to afford a good bearing for the washers or bearing plate of the vertical rods. The purline is usually notched on with sides vertical, as shown. The notches for purlines are generally cut, half in purline and half in the principal, so as to weaken both as little as possible. A cogged joint is stronger and better.

B. AN IRON ROOF.

(257.) *Application to Problem* 5.

Joint A, *Figs.* 95 *and* 96.

These figures give two views of the finished joint.

1. Diameter of Pin. (230.)

It resists the pressure of two rods, each exerting a tensile strain of 15.33 tons, causing single shear.

By the formulæ of (230) $d = .583\sqrt{15.33} = 2.29$ inches, say $2\frac{5}{16}$ inches.

In Figs. 95 and 96, the end A of the truss is supported by the pin, which rests in a semi-cylindrical notch in the top of the cast-iron bed-plate, and which fills the space between the eye-ends of the lower chord. The greatest reaction = maximum pressure of bed-plate against the pin = about 10 tons, and tends to bend the pin by transverse strain, which is uniformly distributed over $2\frac{1}{4}$ inches of the middle of the length of the pin. By the graphical method (56), M = maximum bending moment acting on the pin = 11.25 *inch-tons.*

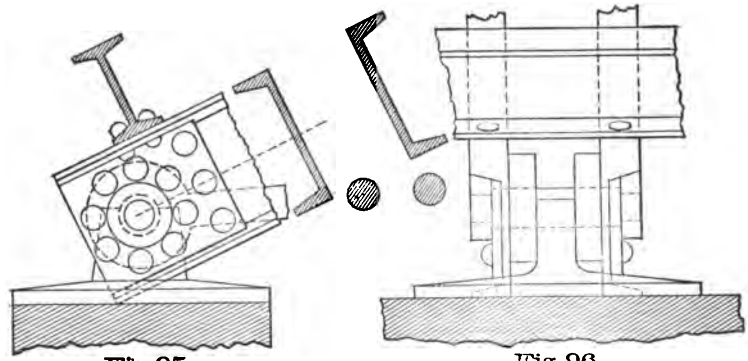


Fig. 95 **Fig. 96**

Joint A.

By formula of (230) $d = 1.107 \sqrt[3]{11.25} = 2.5$ inches.

2. Rod-ends. (231.)

The 2-inch rods composing the member Y1 should have eye-ends of rectangular section, $2 \times 1\frac{5}{8}$ inches on each side of pin, and connected with the body of the rods by long curves instead of those shown in the figures.

3. Joint-plates and Rivets. (229.)

The maximum pressure of each end of the pin against edge of hole through web of channel and the joint-plate = 15.65 tons. The total thickness of the plate and web together $= t = 15.65 \div (6.25 \times 2.5) =$ say 1 inch. As the web is .305 inch thick, we have $1.00 - .305 = .695 =$ say $\frac{11}{16}$ inch = thickness of joint-plate.

If $\frac{3}{4}$-inch rivets are used, the maximum safe resistance of one rivet to single shear $= 2.945 \times .75^2 = 1.655$ tons. (229.) Hence, 1.00 : .695 :: 15.65 : 10.88 tons pressure of pin to be transmitted through the plate and rivets to the channel. And $10.88 \div 1.655 = 6.57$, say 7 rivets are required.

In Fig. 95, 10 rivets are actually employed, those on the left-hand side of the pin only serving to hold the joint-plate and channel firmly together, resisting very little shear.

There should be a nut on each end of the pin to prevent it from slipping endwise. There being very little strain on this nut, the ends of the pin may be reduced as shown, outside the joint-plates, and the nuts may be made cylindrical and of cast iron, being turned up by a bent hook. This will produce some economy of time and space, as the rivets can be set closer to the pin, not interfering with turning up the nut, and the nuts are more easily finished in the lathe.

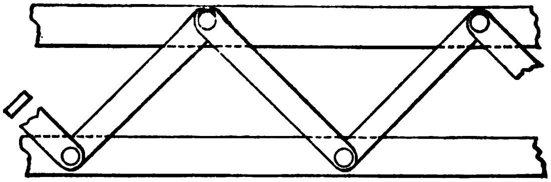


Fig. 97
Lacing Bars.

Or, the screw-thread might be omitted and the nut be held in place by a steel pin driven into a hole drilled through the nut and the end of the pin.

The purline is riveted on as shown, being held by two rivets through each channel. It may sometimes be necessary to strengthen it by a cast-iron plate of proper form riveted in the lower angle between the purline and the principal.

The section of the principal is shown in the same figure.

The bed-plate should be firmly fastened to the wall by four long bolts, at least $\frac{3}{4}$ inch in diameter, and it must be sufficiently large to avoid crushing the wall.

Fig. 97 represents the mode of lacing together the two channels composing the upper chord.

Joint B. Figs. 98 *and* 99.

If the channel bars are not spliced at the joint B, no joint-plate would be necessary; it is here assumed that they are spliced, and that all the pressure of the member X2 against the end of X1 must be transmitted through the joint-plate and the rivets, to allow for imperfect fitting of the ends of the channels against each other.

1. Diameter of Pin.

The pin only resists the pressure of the strut 1 2 = 4.05 tons, causing a single shear of 2.03 tons, and a transverse pressure of 4.05 tons concentrated at its centre, which gives M maximum = 5.82 inch-tons. (56.)

Then by (230) $d = .583\sqrt{2.03} = .833$ inches for shearing.

$d = 1.107\sqrt{5.82} = 1.99$ inches for bending.

Hence, the pin must be 2 inches in diameter.

2. Joint-plates and Rivets.

All the compression in X2 must pass through the plate and rivets under the assumed conditions, determining their dimensions and number. The clear width of the plate is about 5 inches, deducting the diameter of the pin-hole, and it may safely be assumed to resist 5 tons per square inch of cross section.

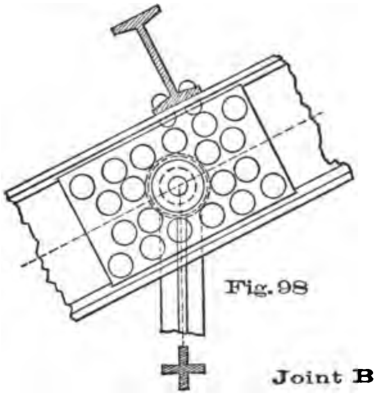


Fig. 98

Joint B.

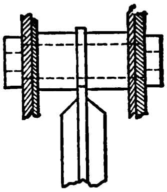


Fig. 99

Then $30.66 \div (2 \times 5) = 3.066$ = sectional area of plate, and $3.066 \div 5 = .613$ = say $\frac{5}{8}$ inch, its thickness.

Using $\frac{3}{4}$-inch rivets, as for joint A, each rivet resists 1.655 tons shear, so that $30.66 \div (2 \times 1.655) = 9.28$, say 10 rivets on *each* end of joint-plate.

Fig. 99 shows a partial cross section of the webs and joint-plates with the pin eye-end of bar, which should be about 1 inch wide on each side of pin, and of the same thickness as the web of the star, with the fillers of cast iron placed between the channels to keep the star in place. These are cylindrical, about $\frac{3}{4}$ inch thick, and are bored for the pin.

Joint D. *Figs.* 100, 101.

1. Diameter of Pin.

Maximum single shear $= 4.97$ tons; M maximum $= 7.14$ inch-tons.

$d = .583\sqrt{4.97} = 1.30$ inches for shearing.

$d = 1.107\sqrt[3]{7.14} = 2.13$ inches, say $2\frac{1}{8}$ inches for bending, which is the required diameter of the pin.

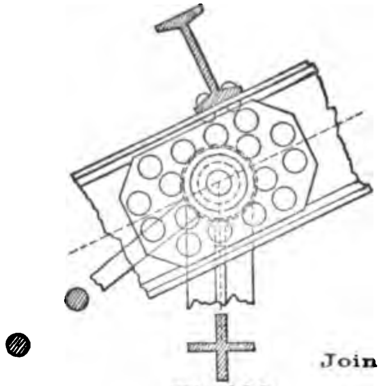


Fig. 100

Joint D.

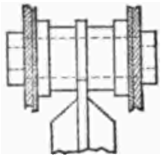


Fig. 101

2. Joint-plate and Rivets.

Clear width about $4\frac{7}{8}$ inches; pressure of X4 $= 24.10$ tons. Hence, $24.10 \div (2 \times 5) = 2.41$ square inches, = area of plate; $2.41 \div 4.875 = .495$, say $\frac{1}{2}$ inch = thickness of plate.

$24.10 \div (2 \times 1.655) = 7.3$, say 8 rivets in *each* end of plates.

3. Eye-ends.

The eye-end on star 3 4 should be 1 inch wide on each side of pin. Those on diagonals 2 3 should be $1\frac{1}{8} \times \frac{7}{8}$. (231.)

Joint F. *Figs.* 102, 103.

1. Diameter of Pin.

Maximum single shear $= 5.42$ tons; M maximum $= 1.95$ inch-tons.

$d = .583\sqrt{5.42} = 1.36$ inches for shear.

$d = 1.107\sqrt[3]{1.95} = 1.38$ inches, say $1\frac{3}{8}$ inches, for breaking.

2. Joint-plates and Rivets.

Net width of plate = about $4\frac{7}{8}$ inches. $16.57 \div (2 \times 5) = 1.66$ square inches; $1.66 \div 4.875 = .33$, say $\frac{3}{8}$ inches, = thickness.

$16.57 \div (2 \times 1.655) = 5.02$, say 5 rivets in each end.

3. Eye-ends.

On star 5 6, 1 inch wide on each side of pin.

On I-post X11, the joint is best made by two plates riveted on and slightly bent, so as to straddle eye-end of star (Fig. 103).

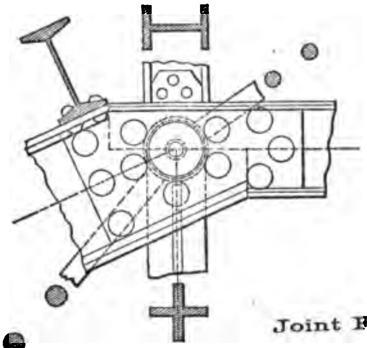


Fig. 102

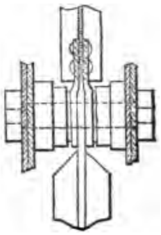


Fig. 103

Joint F.

On diagonal rods 4 5 to be $1\frac{1}{4} \times 1\frac{1}{8}$ inches on each side of pin.

On diagonals 11 10, to be $1\frac{1}{8} \times \frac{7}{8}$ each side of pin.

Joint C. Figs. 104, 105.

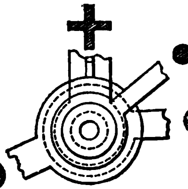


Fig. 104

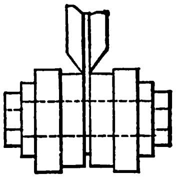


Fig. 105

Joint C.

1. Diameter of Pin.

The maximum single shear is caused by one of the middle pair of rods, composing member Y3, and $= 21.94 \div 2 = 10.97$ tons.

Then $.583 \sqrt{10.97} = 1.93$, say 2 inches diameter of pin.

The joint pins of the lower chord are not supported at their ends, and fail by shearing, rather than by bending or crushing.

2. Eye-end.

On star, 1 inch wide on each side of pin.

On rods Y1, $2 \times 1\frac{1}{2}$ on each side of pin.

On rods Y3, $1\frac{3}{4} \times 1\frac{5}{16}$ on each side of pin.

On rods 2 3, $1\frac{1}{8} \times \frac{7}{8}$ on each side of pin.

Joint E. *Figs.* 106, 107.

1. Diameter of Pin.

Maximum single shear is caused by the middle rods Y3, and = 10.97 tons, as for joint C. Hence, the pin must also be 2 inches.

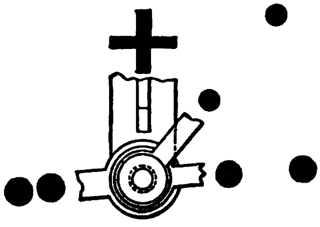


Fig. 106

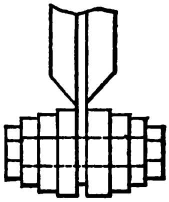


Fig. 107

Joint E.

2. Eye-ends.

On star, 1 inch wide on each side of pin.
On rods Y3, $1\frac{3}{4} \times 1\frac{5}{16}$ on each side of pin.
On rods Y5, $1\frac{1}{2} \times 1\frac{1}{8}$ on each side of pin.
On rods 4 5, $1\frac{1}{4} \times \frac{15}{16}$ on each side of pin.

Joint G. *Figs.* 108, 109.

1. Diameter of Pin.

Maximum single shear is caused by the middle rods composing the member Y7, and $= 16.57 \div 2 = 8.29$ tons.

Then $.583 \sqrt{8.29} = 1.5$ inches.

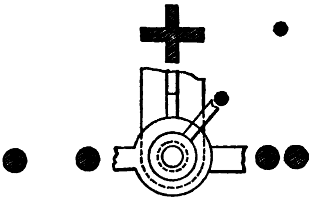


Fig. 108

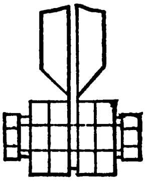


Fig. 109

Joint G.

2. Eye-ends.

On star, 1 inch on each side of pin.
On rods Y5, $1\frac{1}{2} \times 1\frac{1}{8}$ on each side of pin.
On Y7, $1\frac{1}{2} \times 1\frac{1}{8}$ on each side of pin.
On 6 7, $\frac{5}{8} \times \frac{15}{32}$ on each side of pin.

Joint H. Figs. 110, 111.

1. Diameter of Pin.

Maximum single shear is caused by tension on the two rods 11 12, and $= 1.30 \div 2 = .65$ ton.

Then $.583\sqrt{.65} = .47$, say ½ inch.

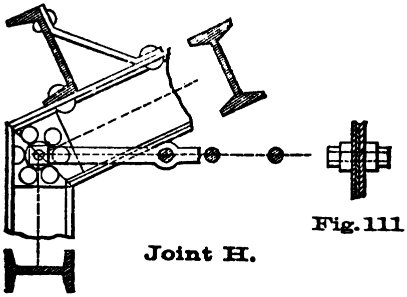


Fig. 111

Joint H.

Fig. 110

2. Eye-ends.

On rods 11 12, these may be 1 × ½ inch, as the size of the rods is determined by the compression acting on them.

The joint-plates should be ¼ inch thick, with 6 ½-inch rivets, as in Fig. 110, the ends of the I-beams being mitred together.

In order to make the joint-pin as short as possible, so as to avoid bending it, the ends of the rods 11 12 are bent inward to fit close against the joint-plates, as in Fig. 111. They are held together and springing is prevented by a rivet or bolt through holes punched just below the member X12, as in Fig. 110.

The channel purline should be stiffened by a wrought-iron stay, riveted to the member X12, as in Fig. 110.

Joint I. Figs. 112, 113.

1. Diameter of Pin.

Maximum single shear is caused by compression on rods 12 13, and $= 22 \div 2 = .11$ ton.

This requires a pin $.583\sqrt{.11} = .194$ inch, but it will be better to make it ½ inch, like the pin at H.

2. Eye-ends.

On rods 12 13, ⅝ × ½ inch on each side of pin. The ends of the rods are bent to fit against the joint-plates, and held together by a rivet, as shown in Figs. 112, 113.

The I-beams should be mitred together and connected by joint-plates ¼ inch thick, with four ½-inch rivets, as in Fig. 112.

The two channel purlines should be connected by a wrought-iron stay riveted to each. Or, a single I-beam might be substituted therefor, though this would probably involve cutting a notch in its lower flange, or chipping away the apex of the truss, to bring the purline to the proper height.

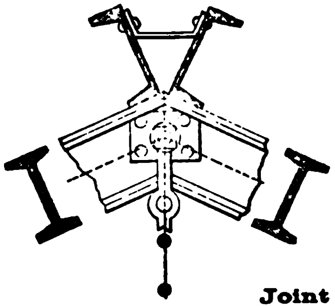


Fig. 112

Joint I.

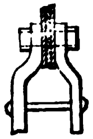


Fig. 113

Joint J. Figs. 114, 115.

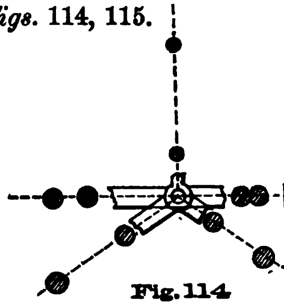


Fig. 114

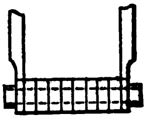


Fig. 115

Joint J.

1. Diameter of Pin.

Maximum single shear is due to the rods 13 14, and $= 1.30 \div 2 = .65$ ton.

$d = .583 \sqrt{.65} = .47$, say ½ inch.

2. Eye-ends.

On rods 10 11, 10 14, 11 12 and 13 14, to be ¾ × ½ inch on each side of pin, as the dimensions of the rods are determined by the compression acting on them.

On rods 12 13, ½ × ½ inch on each side of pin will do.

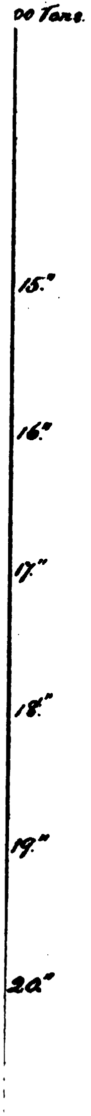
The vertical rods 12 13 are placed nearest the ends of the pin, as in Fig. 115.

The mode of construction here employed for an iron truss is that considered most economical under the special conditions, and for the type of truss selected. A truss of wider span or of different type might require the use of other trade sections of iron, or differently-arranged joints.

Riveted joints are preferred by some engineers, especially in England, but pin joints render the truss more quickly and easily erected, while the axes of the strains in the members more nearly coincide with the axes of the members themselves, unless the riveting is done with great care.

In arranging the details of any joint, it should always be remembered that its least resistance is always the limit of its strength, so that all its parts should be of equal strength, so far as possible.

❧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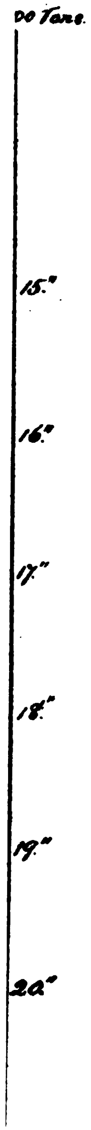
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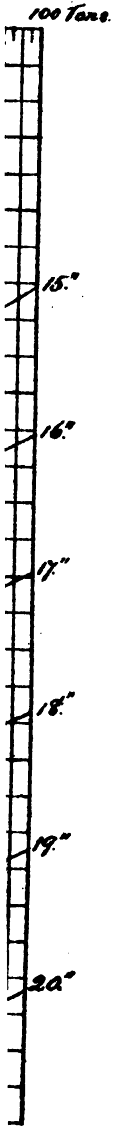
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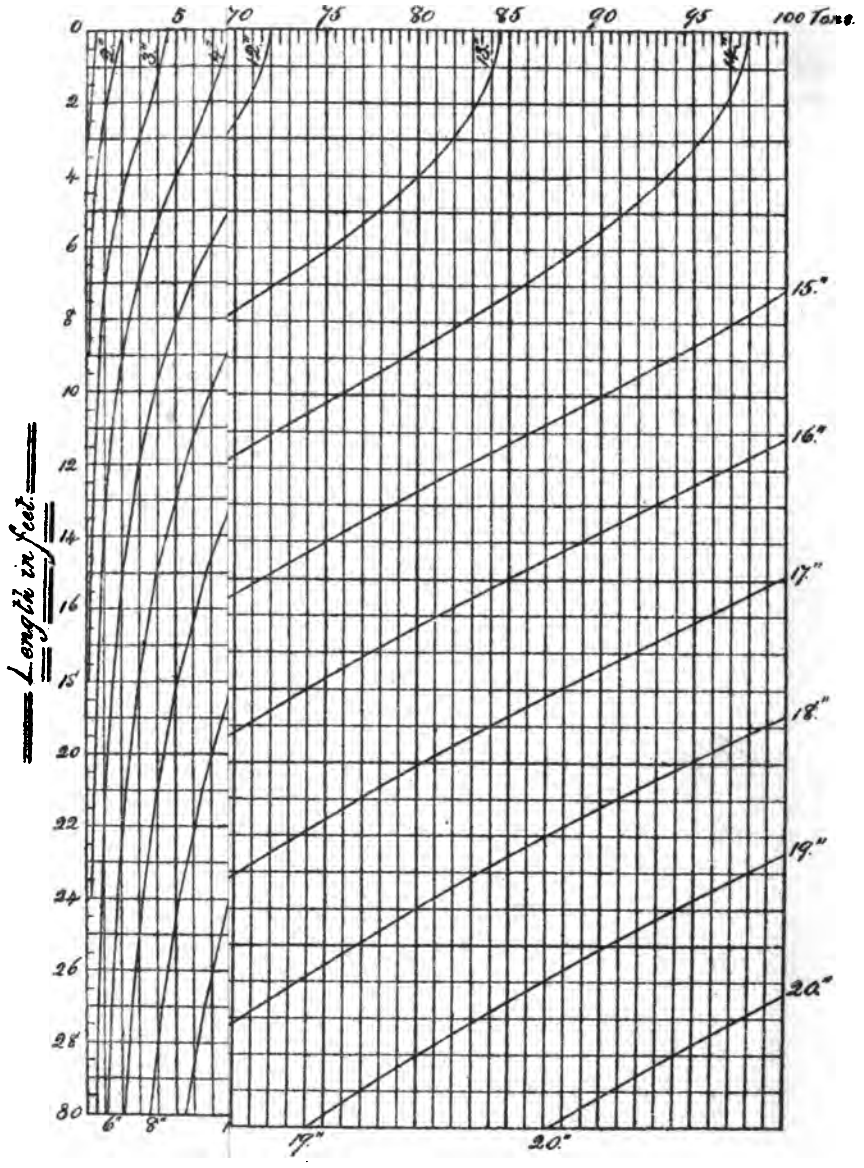
Length in feet.
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2
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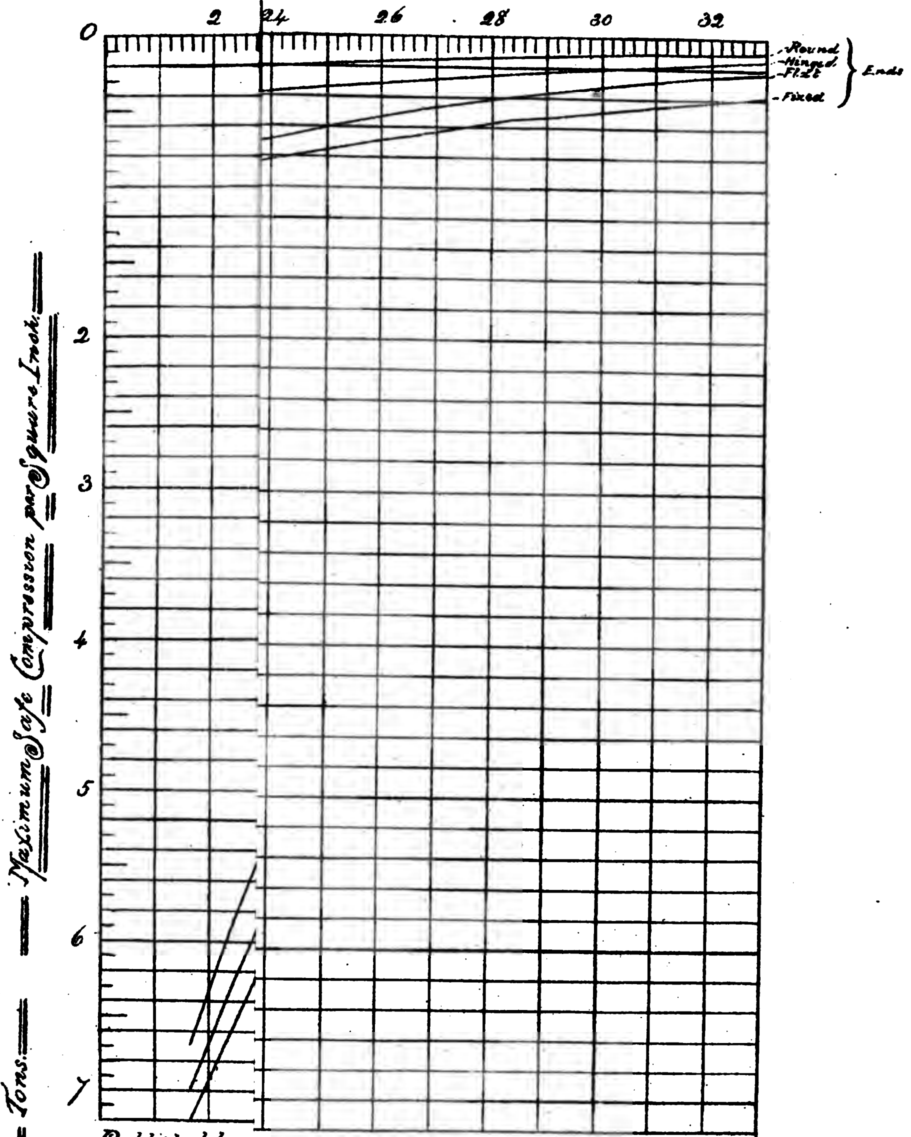
6
8
10
12
14
16
15'
20
22
24
26
28'
80
5
70
75
80
85
90
95
100 Tons.
2"
3"
4"
12"
13"
14"
15."
16"
17"
18."
19."
20."
6"
8"
17"
20."

Table 4



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